Asset Transfer Measurement Rules*

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Abstract

We study the design of measurement rules when banks engage in loan sales in secondary credit markets. Our model incorporates two standard frictions: 1) banks' monitoring incentives decrease in loan transfers, and 2) banks have private information about loan quality. Under only the monitoring friction, we find that the optimal measurement rule sets the same measurement precision regardless of bank characteristics, and strikes a balance between disciplining banks' monitoring efforts vs. facilitating efficient risk sharing. However, under both frictions, uniform measurement rules are no longer optimal but induce excessive retention, thus inhibiting efficient risk sharing. We show that the optimal measurement rule is contingent on the amount of loan retained. In particular, measurement occurs if and only if the bank retains a sufficiently large proportion of its loan portfolio and whenever measurement occurs, the precision of the measurement rule increases in the proportion retained. We relate our results to current regulation for asset transfers.

Keywords: Asset Transfer Measurement Rules; Loan Sale; Securitization; Credit Risk Transfer; Financial Institutions.

JEL codes: G21, G28, M41, M48.

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1 Introduction

Credit risk transfers are quite pervasive in practice by playing a key economic role for banks' activities (Pozar et al, 2010). Current accounting standards on asset transfers focus on control as a key criterion in determining the appropriate accounting treatment of such transfers. (ASC 860, Financial Accounting Standards Board). However, the notion of control is elusive unless the incentives of transferors are well understood. Indeed, a central motive for amending the accounting standards on asset transfers to their current form lies in the concerns that prior accounting standards could distort the incentives of asset transferors. Robert Herz, the former chairman of the FASB, posited the changes as necessary to "improve existing standards and to address concerns about companies who were stretching the use of off-balance sheet entities to the detriment of investors." (FASB, 2009)[1] Our goal in this paper is to ask a more primitive question: given the frictions faced by asset transferors, how should measurement rules be designed to maximize asset transfer efficiency?

We model a representative risk-averse bank that chooses how much credit risk to transfer by selling a proportion of its loan portfolio to a secondary credit market. The bank faces two standard frictions. First, its incentives to monitor borrowers diminish as a result of selling its loan portfolio. Second, it has private information about the quality of its loan portfolio but such information cannot be credibly disclosed to outsiders. Both costly monitoring and/or higher loan quality stochastically improve the terminal payoffs of the loan portfolio. Given the monitoring and informational frictions, we investigate how measurement rules

[1]The amendment includes "requir(ing) more information about transfers of financial assets, including securitization transactions, and where companies have continuing exposure to the risks related to transferred financial assets. It eliminates the concept of a 'qualifying special-purpose entity,' changes the requirements for derecognizing financial assets, and requires additional disclosures."

should be designed to maximize *ex ante* surplus. We study two types of measurement rules: a *uniform* measurement rule that requires the same measurement precision of the loan's terminal payoffs regardless of banks' characteristics and a *contingent* measurement rule that makes the precision of the measurement rule contingent on banks' observable characteristics. In practice, a contingent measurement rule may be written on any observable characteristics of banks' environments. We focus on the level of loan retention as it may serve as an indicator of control which is a central criterion for measurement under current accounting standards for asset transfers.

To develop intuition for our main model, we first study a benchmark setting in which the bank's monitoring effort is unobservable but its loan quality is publicly known. We show that while both loan retention and a more precise measurement rule enhance monitoring incentives, they also impede risk-sharing incentives. In particular, a higher loan retention and a more precise measurement rule are substitutes in providing monitoring incentives but are complements in providing risk-sharing incentives. More importantly, we show that the optimal measurement rule is decoupled from the optimal retention choices so that one should set a measurement rule that is independent the bank's optimal retention choices. Such a uniform measurement rule trades off the benefit of providing efficient monitoring incentives vs. facilitating the efficient transfer of risk. Under such a uniform rule, we show that measurement is desirable if and only if the efficiency loss from the reduction in risk-sharing is not too high and/or the bank's monitoring incentives are sufficiently high. The intuition behind this result is straightforward. While measurement inhibits risk-sharing due to the well-known Hirschleifer effect (Hirschleifer, 1971), it also provides efficient monitoring incentives by increasing the sensitivity of prices to banks' fundamentals. Consequently, measurement is more likely in those environments in which gains from efficient monitoring overwhelm losses from inefficient risk-sharing.

When loan quality is unobservable, besides its monitoring role, the proportion of loan retention acquires an additional informational role. The latter role, in turn, induces banks to increase loan retention in order to credibly communicate their private information to outsiders. Such excessive retention is inefficient and reduces the bank's surplus. More interestingly, we show that a uniform measurement rule is no longer optimal because a bank's asset transfer policy and the measurement rule are now intertwined. In particular, when risk-sharing considerations are sufficiently strong, measurement exacerbates the over-retention problem arising from the adverse selection problem and reduces efficiency. By tailoring the measurement rule to the proportion of loan retention, one may improve surplus by influencing banks' retention policy.

Our main result is that the optimal measurement rule takes the form of a contingent measurement rule such that measurement precision depends on the bank's asset transfer policy. Under the contingent measurement rule, measurement occurs if and only if the bank retains a sufficiently large proportion of its loan portfolio and whenever measurement occurs, the precision of the measurement rule increases in the proportion retained. Given that the bank's asset transfer policy, in equilibrium, depends on the exogenous parameters of the bank's environment, these exogenous parameters also determine when measurement is more likely to be occur under the optimal rule. We find that measurement should always occur when monitoring considerations are relatively more important than risk-sharing considerations. But when risk-sharing considerations are sufficiently important, measurement is optimal if and only if the loan quality of the bank's portfolio is sufficiently high.

As mentioned above, we do not explicitly model control issues that seem to be the focus of standard setters. However, our optimal measurement rule–that measurement precision and asset transfer decisions are inherently linked– provides a useful benchmark for understanding how the extent of credit risk transfer should be factored into judgements about how such loans should be measured. In particular, according to the contingent measurement rule, no measurement is optimal if and only if the bank has transferred most of its loans. To the extent that a bank that has transferred most of its loans would have relatively little control over the loans, our contingent measurement rule provides theoretical support for adopting "control" as the key guiding principle under current asset transfer measurement rules. Furthermore, our comparative statics provide some testable predictions about environments in which measurement is more likely to be optimal.

Our analyses also have implications for the Dodd-Frank Act that requires securitization sponsors to retain not less than a 5% share of the aggregate credit risk of the assets they securitize. Our results suggest that this one-size-fits-all risk retention requirement is suboptimal: optimal risk retention level should vary according to the riskiness of the underlying assets and the information environment of banks.

1.1 Related literature

Our model combines important features of the classical models of Leland and Pyle (1977) and Kanodia and Lee (1998). Similar to Leland and Pyle, we also show that the bank's loan transfer decision conveys information about loan quality inducing the bank to transfer fewer loans. However, our model differs from that of Leland and Pyle in two important ways. First, we consider loan retention decision in the presence of both private information and unobservable monitoring. Second, there are no measurement issues in Leland and Pyle. In our study, we derive the optimal measurement rule that plays a key role in influencing a bank's loan retention policy. As in Kanodia and Lee, a more precise measurement rule in our environment disciplines *ex-ante* monitoring incentives but also destroys *ex-post* risk-sharing. However, Kanodia and Lee do not investigate asset transfer decisions and therefore cannot study the interaction between measurement rules and asset transfer policies which is our main focus.

Our study is also related to a large banking literature on credit risk transfers. Early work such as Greenbaum and Thakor (1987) investigate a bank's choice of whether to fund the loans it originates by either issuing deposits or by selling loans to investors. They show that higher quality loans will be sold while lower quality loans will be funded via deposits. Pennacchi (1988) considers a model where banks may improve the returns on loans by monitoring borrowers. He shows that by designing the loan sales contract in a way that gives the bank a disproportionate share of the gains to monitoring, a greater share of the loan can be sold and, hence, a greater level of bank profits can be attained. Gorton and Pennacchi (1995) study a model of incentive-compatible loan sales that allows for implicit contractual features between loan sellers and loan buyers. They theoretically and empirically show that, by maintaining a portion of the loan's risk, banks convince loan buyers of its commitment to evaluate the credit of borrowers. DeMarzo and Duffie (1999) consider the optimal design of an asset-backed security and analyze a trade-off between the retention cost of holding cash flows, and the liquidity cost of including the cash flows and making the security design more sensitive to the issuer's private information. Allen and Carletti

(2006) develop a model of how credit risk transfer affects contagion. Using a model with banking and insurance sectors, they show that credit risk transfer is beneficial when banks face uniform demand for liquidity but when they face idiosyncratic liquidity shock, credit risk transfer can increase contagion. Parlour and Plantin (2008) develop a model in which banks receive either proprietary information about loan quality or a shock to their discount rate. Either effect induces banks to transfer credit risk resulting in an adverse selection problem. Parlour and Plantin investigate when such credit risk markets arise and whether this is efficient. Our work is related to these prior studies because either private information, or monitoring or risk-sharing concerns is an important force that affects asset transfers in all these studies. However, none of them study measurement issues which is our main focus. An exception is Goldstein and Leitner (2018) who develop a model in which disclosure can destroy risk-sharing opportunities for banks but some level of disclosure is necessary for risk sharing to occur. However the focus of their study differs significantly from ours. They study the optimal disclosure policy of a regulator who has information about banks as a result of conducting stress tests. We investigate how optimal measurement rules should be designed to affect banks' retention decisions in the presence of both moral hazard and adverse selection problems.

More broadly, our focus on measurement rules on asset transfer policies connects our paper to the literature on the role of accounting measurement in the financial industry. Corona, Nan, and Zhang (2014) examine how improving the quality of accounting information affects the efficiency of capital requirements and banks' risk-taking incentives, taking into account the competition among banks. Corona, Nan, and Zhang (2019) examine the coordination role of stress-test disclosure in affecting bank risk-taking. Gao and Jiang (2018), Zhang (2019), and Liang and Zhang (2019) study how different aspects of accounting measurements may help to stabilize bank runs. Corona, Nan, and Zhang (2019) and Bertomeu, Mahieux, and Sapra (2022) study the joint efficacy of capital requirement policy and accounting measurement rules in disciplining banks' risk-taking and stimulating bank lending.[2] Lu, Sapra, and Subramanian (2019) study the optimal use of mark-to-market accounting in implementing capital requirements, in the presence of asymmetric information and agency conflicts. Our study is more closely related to Bleck and Gao (2017) who analyze banks' loan origination and retention decisions in a signalling model. However, unlike our study, they do not examine the *ex ante* optimal measurement rule but instead focus on characterizing the economic consequences of two specific measurement rules: mark-to-market versus historical cost. They find that, compared to historical cost, mark-to-market accounting improves the accuracy of loan valuation, forces good banks to retain more risk on their balance sheet, and can reduce banks' origination efforts.

Finally, the empirical accounting literature provides evidence on how the accounting for securitization may have economic consequences on firms. Dechow and Shakespeare (2009) investigate whether firms exploit the accounting treatment for securitization to burnish their financial statements. Barth, Omarzabal, and Taylor (2012) show that credit-rating agencies and the bond market differ in their assessments of credit risk transfers in terms of how they evaluate retained vs. non-retained interests of securitized assets. More recently, Dou, Ryan, and Xie (2018) provide evidence on how recent accounting standards that tightened the accounting for securitization and consolidations have real effects on banks' mortgage

[2] Mahieux (2019) studies the joint efficacy of capital requirements and accounting rules in the specific context of fair value accounting.

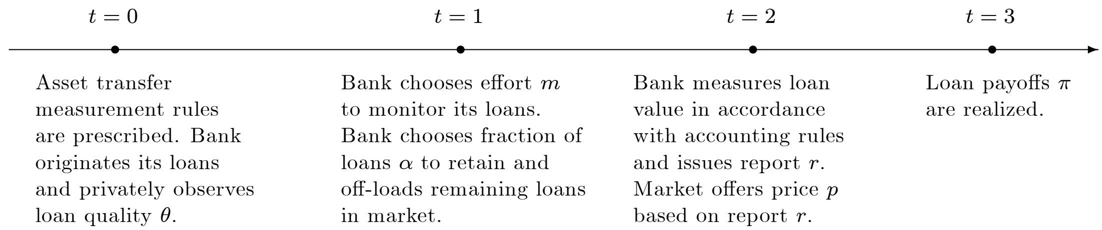

Figure 1: Timeline of the model

approval and sale rates. We do not focus on the specifics of the accounting standards for transfers in our model. Instead, we investigate how, given retention policies, measurement rules should be designed to influence *informational* features of accounting reports, and relate the implications of those measurement rules to the current standards for asset transfers.

The remainder of the paper is structured as follows. Section 2 describes the model. Section 3 analyzes the model. Section 4 concludes. An Appendix contains the proofs of the major results.

2 The Model

2.1 Timing of events

We examine an environment that consists of a representative bank owner (henceforth, bank) and a secondary loan sale market. The bank has an additive and separable utility function with constant absolute risk aversion $\tau > 0$. The discount factor is normalized to 1. Figure 1 summarizes the timing of events.

At date $t = 0$, the bank is endowed with a portfolio of loans originated earlier. At the terminal date, $t = 3$, the loan portfolio generates stochastic terminal cash flows, $\pi(\theta, m)$, that depend on both the credit quality θ of the loan portfolio and the bank's *ex-post* effort

m to monitor borrowers. For simplicity, we assume that

$$\pi = \theta + m + \eta. \tag{1}$$

The loan quality $\widetilde{\theta}$ has a distribution $F(.)$ and a density $f(.)$ with full support on $\left[\underline{\theta}, \bar{\theta}\right]$. The random variable $\widetilde{\eta}$ follows a normal distribution with mean 0 and precision h_η. Equation (1) implies that either a higher loan quality or a greater monitoring effort improves the performance of the loan portfolio in the sense of first-order stochastic dominance. The shock $\widetilde{\eta}$ captures the residual uncertainty regarding the loans' cash flows for a given level of monitoring effort and loan quality. We assume that the bank learns the quality θ of its loan portfolio privately in the process of loan origination and such information cannot be credibly disclosed to outsiders.[3]

After originating the loan portfolio, i.e., at $t = 1$, the bank chooses unobservable effort $m > 0$ to monitor borrowers at a private cost of $\frac{c}{2}m^2$, where $c > 0$. In addition, since the bank is risk averse, it has an incentive to engage in credit risk transfer by selling a portion of its loan portfolio in the secondary loan sale market. In particular, the bank chooses the fraction $\alpha \in [0, 1]$ of the loan portfolio to retain and therefore the fraction $(1 - \alpha)$ to sell. We assume that the proportion α is publicly observable.

Before the terminal payoffs π of the loan portfolio are realized, at $t = 2$, the bank issues an accounting report r that measures π. We adopt the following specification of the report

$$r = \pi + \varepsilon. \tag{2}$$

[3] The assumption that banks have private information about borrower default risk is consistent with research by Dahiya, Puri and Saunders (2003) and Marsh (2006).

The measurement noise $\widetilde{\varepsilon}$ is normally distributed with mean 0 and precision h_{ε}. A key focus of our study is to derive the optimal *ex-ante* asset transfer measurement rules that govern the measurement process and the informational features of the accounting report r. To this end, we consider and compare two types of measurement rules: a *uniform* measurement rule that does not depend on observable bank characteristics and hence is the same across all banks and a *contingent* measurement rule that sets measurement precision based on observable bank characteristics. One such bank characteristic in our model is the proportion α of the loan portfolio that the bank retains. Hence under a uniform measurement rule, the required measurement precision is independent of the fraction of loan retention α whereas under a contingent rule, the measurement precision is designed to be a function of α.

After releasing the report, the bank sells or equivalently transfers without recourse a fraction $1-\alpha$ of its loan portfolio at a per-unit transfer price p.[4] In practice, the secondary loan sale market is often illiquid and transaction prices can be highly sensitive to liquidity effects. In order to account for this illiquidity of the loan portfolio, following Plantin, Sapra and Shin (2008), we assume that the loan sale price p is given by:[5]

$$p = E\left[\widetilde{\pi}|r\right] - \delta(1-\alpha), \tag{3}$$

where $E\left[\cdot\right]$ denotes the expectations operator, $1-\alpha$ is the portion of loans that are sold, and $\delta \geq 0$ is a parameter that captures the liquidity of the loans. The loan price depends on both

[4] Throughout the paper, we use the terms loan sales and loan transfers interchangeably. By loan transfers, we therefore mean loan transfers without recourse.

[5] Our main result holds qualitatively when the loan market is fully competitive and liquid, i.e., $\delta = 0$ and the price equals the expected terminal loan cash flows. We introduce the liquidity discount δ to capture a key institutional feature of the secondary loan sale market and to generate further empirical and policy implications.

the expected terminal cash flows of the loan portfolio $E[\widetilde{\pi}|r]$ conditional on the report and the liquidity of the loans. When $\delta = 0$, the loan market is infinitely liquid so that the price of the loan equals its terminal cash flow in expectation. When $\delta > 0$, the price decreases in the amount of loans sold. The larger δ is, the more illiquid the loan market is and the more sensitive the selling price is to the loan sale amount. In this light, the price effect of loan sales captures the risk-bearing capacity of the secondary loan sale market to absorb loans that banks off-load.

At $t = 3$, the terminal payoffs π of the loan portfolio are realized.

2.2 Payoffs

As a preliminary analysis, we specify the bank's payoff and its *ex ante* surplus. The bank obtains payoffs at three dates. At $t = 1$, the bank incurs a private monitoring cost of $\frac{c}{2}m^2$. At $t = 2$, the bank receives a price of $(1-\alpha)\,p$ from transferring the loans in the loan sale market. We will verify that, given the loan quality θ, the equilibrium price p is normally distributed. Therefore, using standard results in finance, the bank's expected utility of the date-2 consumption can be represented as

$$(1-\alpha)\,E[\widetilde{p}|\theta] - \frac{\tau}{2}(1-\alpha)^2\,Var(\widetilde{p}|\theta)\,. \tag{4}$$

Note that since the bank is risk averse, higher price volatility reduces the bank's payoffs. At $t = 3$, the bank receives a terminal cash flow from the loans it retains, $\alpha\pi$. Since π is normally distributed given θ, the bank's expected utility of the date-3 consumption can be represented as

$$\alpha E\left[\widetilde{\pi}|\theta\right] - \frac{\tau}{2}\alpha^2 Var\left(\widetilde{\pi}|\theta\right). \tag{5}$$

In sum, since the bank's utility is additive and separable, its total payoff is the sum of its expected payoffs at the three dates $t \in \{1, 2, 3\}$, i.e., for a given θ, the bank chooses its monitoring effort m and the asset transfer decision α to maximize

$$U(m, \alpha; \theta) = E\left[(1-\alpha)\widetilde{p} + \alpha\widetilde{\pi}|\theta\right] - \frac{\tau}{2}\left[\alpha^2 Var\left(\widetilde{\pi}|\theta\right) + (1-\alpha)^2 Var\left(\widetilde{p}|\theta\right)\right] - \frac{c}{2}m^2. \tag{6}$$

Lastly, the *ex-ante* surplus equals the bank's *ex-ante* expected utility

$$W = E_\theta\left[U\left(m, \alpha; \widetilde{\theta}\right)\right] = \int_{\underline{\theta}}^{\overline{\theta}} U(m, \alpha; \theta) f(\theta) d\theta. \tag{7}$$

2.3 Assumptions

We now motivate some key ingredients of our model.

First, we assume the bank acts in a risk averse manner in order to induce the bank to engage in credit risk transfers via loan sales in the secondary market. Credit risk transfers play a key economic motive for banks' activities of loan sales and securitization in practice. For instance, Pozar et al (2010), in discussing the efficiency gain of securitization, argue that "securitization involving real credit risk transfer is an important way for an issuer to limit concentrations to certain borrowers, loan types and geographies on its balance sheet." In this light, outside investors may be better suited than loan originators in bearing loan risk, because investors often hold a broadly diversified portfolio of assets. Similarly, Stein (2010) argues "(w)hen banks sell their loans into the securitization market, they distribute the risks associated with these loans across a wider range of end investors, including pension funds, endowments, insurance companies, and hedge funds, rather than taking on the risks entirely themselves. This improved risk-sharing represents a real economic efficiency and lowers the ultimate cost of making the loans." Interestingly, Goldstein and Leitner (2018) show that risk-neutral banks can still behave in a risk-averse manner due to the liquidity needs of their consumers. Stated differently, liquidity needs induces risk aversion even in the presence of fundamental risk neutrality.[6]

Second, our focus is on how asset transfer measurement rules affect the informational features (i.e., measurement precision) of accounting reports. Consequently, we model the simplest form of credit risk transfer: loan transfers without recourse. In practice, credit risk transfers may involve complex security designs such as securitization and/or the use of credit derivatives that specify the rights of transferors and transferors under various contingencies.[7] In that sense, we cannot directly speak to control issues that seem to be the focus of standard setters. However, by focusing on loan transfers without recourse, we believe that our framework provides a simple but important benchmark for understanding how the proportion of loans transferred should be factored into judgements about how such loans should be measured. To the extent that a bank that has transferred most of its loans would have relatively little control over the loans, our examination of how measurement should be

[6] Alternatively, we could assume a risk-neutral bank receiving shocks to its discounting factor (i.e., shocks to the opportunity cost of carrying outstanding loans) as in Parlour and Plantin (2008), which also creates a demand for loan sales.

[7] Loan sales and securitizations are the two main types of credit risk transfers. A loan sale merely transfers a part of the ownership of a loan portfolio to others, whereas securitization alters patterns of cash flows and other asset properties. Furthermore, most loan sales are made without recourse and, unlike securitization, there are usually no explicit credit enhancement (Greenbaum et al, 2019).

made contingent on the proportion of asset transfer provides theoretical support for adopting "control" as the key guiding principle under current asset transfer measurement rules.

3 Analysis

3.1 Observable loan quality

We start the analysis by assuming that loan quality θ is publicly observable. We solve the model using backward induction. At $t = 2$, the market offers a price, given in (3), that depends on both the expected terminal cash flows π of the loan portfolio conditional on the report r and the liquidity of the loan portfolio. Since terminal cash flows π depend on the bank's unobservable monitoring effort, the market rationally forms a conjecture $\hat{m}$ about the bank's monitoring effort in order to use the report to update its beliefs about π. Of course, this conjecture must be correct in equilibrium. Given $\hat{m}$, the distribution of π conditional on r is normal so that the price for loans is given by

$$p^* = E\left[\widetilde{\pi}|r, \hat{m}\right] - \delta(1-\alpha) = \beta r + (1-\beta)\left(\theta + \hat{m}\right) - \delta(1-\alpha). \tag{8}$$

The price depends on a weighted average of the accounting report r and the prior expectation about π, given the conjecture about the monitoring effort. The weight $\beta \equiv \frac{h_\varepsilon}{h_\varepsilon + h_\eta}$ placed on the report r is strictly increasing in and is isomorphic to the measurement precision h_ε. Note that $\beta = 0$ corresponds to the case of no measurement whereas $\beta = 1$ corresponds to the case of perfect measurement. For expositional convenience, we hereafter refer to β as the measurement precision of the report.

We next solve for the bank's choice of monitoring effort m at $t = 1$. Substituting the transfer price (8) into the bank's payoff (6) and rearranging terms yields

$$U(m, \alpha; \theta) = \theta + \underbrace{[\alpha + (1-\alpha)\beta] m - \frac{c}{2} m^2}_{\text{gain from monitoring}} + (1-\alpha)(1-\beta)\hat{m} - \underbrace{\frac{\tau}{2h_\eta}\left(\alpha^2 + (1-\alpha)^2 \beta\right)}_{\text{loss from inefficient risk sharing}} - \delta(1-\alpha)^2. \tag{9}$$

From (9), it follows that choosing both the proportion of loan retention and the precision of measurement of loan performance results in a trade-off: they both provide monitoring incentives that increase the bank's payoffs but they simultaneously inhibit efficient risk-sharing that reduces the bank's payoffs. Furthermore, the last term of (9) captures the liquidity cost borne by the bank when transferring loans in the secondary market. The more loans the bank transfers, the greater the liquidity cost.

To see the monitoring roles of loan retention and measurement, differentiating (9) with respect to m yields the *equilibrium* monitoring effort m^* that satisfies

$$\alpha + (1-\alpha)\beta = cm^*. \tag{10}$$

The right-hand side of (10) represents the marginal cost of monitoring, while the left-hand side represents the marginal benefit of monitoring. The left-hand side captures the often-debated incentive problem when banks transfer loans (e.g., Keys et al, 2010): their incentives to monitor their loans are lower whenever they have "less skin in the game." To see this more clearly, note that when $\beta = 0$, m^* decreases as the proportion of loan retention α decreases. However, in the presence of measurement, i.e., when $\beta > 0$, (10) also suggests that higher precision of measuring loan performance (i.e., a larger β) is a *substitute* for loan retention in incentivizing the bank to monitor. More precise measurement improves the efficiency of pricing the loans in alignment with their underlying cash flows, thereby disciplining the bank.

Similarly, to see their risk-sharing roles, we next derive the bank's loan transfer decision α and the optimal measurement precision β that maximizes the *ex-ante* surplus (7). Note that, when the loan quality θ is observable, implementing either a uniform measurement rule or a rule contingent on α does not make a difference. This is because, in this case a uniform rule essentially makes the measurement precision β independent of α whereas a contingent rule allows β to be set after observing α. However, the order regarding the choices of α and β does not matter because, absent any private information about θ, the bank's payoff coincides with the *ex-ante* surplus, thereby making the uniform and the contingent rules equivalent. To see the algebraic equivalence, substitute the equilibrium monitoring effort specified in (10) into (9) to get

$$U\left(m^{*},\alpha;\theta\right)=\theta+m^{*}-\frac{c}{2}\left(m^{*}\right)^{2}-\frac{\tau}{2h_{\eta}}\left(\alpha^{2}+(1-\alpha)^{2}\beta\right)-\delta(1-\alpha)^{2}. \tag{11}$$

Note that we have imposed the rational expectation requirement that the market's conjecture is consistent with the equilibrium, i.e., $\hat{m}=m^{*}$.[8] In addition, from (7), the *ex-ante* surplus

$$W\equiv\int_{\underline{\theta}}^{\overline{\theta}}U\left(m,\alpha;\theta\right)f(\theta)d\theta=E(\widetilde{\theta})+m^{*}-\frac{c}{2}\left(m^{*}\right)^{2}-\frac{\tau}{2h_{\eta}}\left(\alpha^{2}+(1-\alpha)^{2}\beta\right)-\delta(1-\alpha)^{2}. \tag{12}$$

Note that W differs from $U\left(m^{*},\alpha;\theta\right)$ only by the constants θ and $E\left(\widetilde{\theta}\right)$. Therefore, the pair

[8] Note that given $\{\alpha,\beta,c\}$ are common knowledge, the market can perfectly conjecture m from (10).

of $\{\alpha, \beta\}$ that maximizes the bank's payoff $U(m^*, \alpha; \theta)$ also maximizes the *ex-ante* surplus W. In the next lemma, we formally state the equivalence result between the uniform and the contingent measurement rules when private information is absent.

Lemma 1 *When the loan quality θ is publicly observable, the equilibrium outcomes under the uniform measurement rule coincide with those under the contingent measurement rule.*

A direct implication of Lemma 1 is that, since the bank's objective function is isomorphic to the *ex-ante* surplus, we only need to solve for the pair of $\{\alpha, \beta\}$ that maximizes (12) to completely characterize the equilibrium. Differentiating (12) with respect to β and rearranging terms yields

$$(1 - cm^*) \frac{\partial m^*}{\partial \beta} = \frac{\tau}{2h_\eta}(1-\alpha)^2. \tag{13}$$

Equation (13) characterizes the optimal trade-off in setting the measurement precision. The left-hand side of (13) captures the marginal benefit of improving measurement precision in disciplining the bank's monitoring effort.[9] The right-hand side captures the marginal cost of more precise measurement in inhibiting the efficient transfer of risk. Recall that transferring the loans from the bank to the market improves efficiency because the market is better at absorbing risk than the risk-averse bank. Such risk transfer is best achieved if the transfer price is insensitive to the performance of the loan portfolio so that the risk-averse bank bears no risk after the transfer. However, more precise measurement makes the price more sensitive to the terminal payoffs so that the bank faces higher price volatility even after off-loading its loans. Such volatility constitutes a cost of measurement.

[9] The left-hand side of (13) is strictly positive because, from (10), $\frac{\partial m^*}{\partial \beta} = \frac{1-\alpha}{c} > 0$ and $1 - cm^* = (1-\alpha)(1-\beta) > 0$. Thus the left-hand side of (13) is given by $\frac{(1-\alpha)^2}{c}(1-\beta)$.

Interestingly, the right-hand side of (13) decreases in α which, in turn, suggests that–unlike their monitoring roles discussed above where they are substitutes–higher precision of measurement and loan retention are *complements* when it comes to risk-sharing in the sense that the risk-sharing loss from measurement is lower when the bank has retained more loans, and *vice versa*. Intuitively, the price volatility induced by measurement is lower the larger the proportion of the loan portfolio that the bank retains. Accordingly, measurement is least costly to the bank from a risk-sharing perspective if the bank has retained a large proportion of its loan portfolio.

Analogously, differentiating (12) with respect to α and rearranging terms yields[10]

$$(1 - cm^*) \frac{\partial m^*}{\partial \alpha} + 2\delta(1 - \alpha) = \frac{\tau [\alpha - (1 - \alpha)\beta]}{h_\eta}. \tag{14}$$

Equation (14) illustrates the trade-offs the bank makes in its asset transfer decision: while transferring more assets improves risk sharing, it also weakens the bank's incentive to monitor in addition to the liquidity cost the bank incurs when selling loans in the illiquid secondary market. Moreover, the right-hand side of (14) once again illustrates the complementarity between measurement and loan retention in affecting the risk-sharing loss. Given measurement, loan retention has a smaller adverse effect on risk sharing and the adverse effect gets even smaller as the proportion of the loan portfolio transferred increases.[11] This is because, conditional on measurement, the bank would still incur the risk-sharing loss even if it had transferred most of its loans.

[10] The left-hand side of (14) is strictly positive because, from (10), $\frac{\partial m^*}{\partial \alpha} = \frac{1-\beta}{c} > 0$ and $1 - cm^* = (1 - \alpha)(1 - \beta) > 0$. Thus the first term on the left-hand side of (14) is given by $\frac{1-\alpha}{c}(1 - \beta)^2$.

[11] Mathematically, the right-hand side of (14) decreases in β and the decrease is proportional to $(1 - \alpha)$.

Solving equations (13) and (14) yields the equilibrium levels of loan retention and measurement precision. We formally state the equilibrium in the following proposition.

Proposition 1 *When loan quality θ is publicly observable, the optimal choices $\{\alpha_0, \beta_0\}$ of loan retention and measurement precision are, respectively,*

$$\begin{cases} \alpha_0 = \frac{8\delta h_\eta^2 + 4h_\eta\tau - c\tau^2}{8\delta h_\eta^2 + \tau(8h_\eta - c\tau)} \text{ and } \beta_0 = 1 - \frac{c\tau}{2h_\eta} & \text{if } \frac{\tau}{2h_\eta} < \frac{1}{c}; \\ \alpha_0 = \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau} \text{ and } \beta_0 = 0 & \text{if } \frac{\tau}{2h_\eta} \geq \frac{1}{c}. \end{cases} \tag{15}$$

Proposition 1 is intuitive and illustrates that, absent any private information, measurement rules and asset retention decisions are decoupled in the sense that they do not depend on each other. Instead, the optimal measurement rule sets the same measurement precision for all banks regardless of their loan quality θ. Moreover, the measurement rule requires measuring the value of transferred loans if and only if monitoring considerations are more important relative to risk-sharing considerations. In particular, $\beta_0 > 0$ whenever $\frac{\tau}{2h_\eta} < \frac{1}{c}$, i.e., when banks have sharp incentives to monitor loans (i.e., c is low) but the bank's risk aversion τ is low and the loan's terminal cash flows are less volatile, i.e., h_η is high. Accordingly, upon measurement, the precision should be higher, i.e., β_0 increases, when either c decreases and/or $\frac{\tau}{h_\eta}$ decreases. Conversely, as risk considerations become more important, i.e., $\frac{\tau}{h_\eta}$ increases but monitoring incentives become less sharp, i.e., monitoring cost c increases, the cost of measurement in inhibiting risk transfer increases relative to the benefit of measurement in disciplining the bank. In that case, the optimal measurement rule calls for less and less precise measurement, i.e., β_0 decreases to zero.

3.2 Unobservable loan quality

We now analyze the complete model in which the loan quality θ is privately known by the bank. In the presence of private information, the bank's objective function no longer coincides with the bank's *ex-ante* surplus. Therefore, the equilibrium outcome under the uniform measurement rule may now differ from that under the contingent rule. We examine the equilibrium outcome under each of the two rules separately.

3.2.1 Uniform measurement rule

We start with the uniform measurement rule in which the measurement precision β is the same regardless of banks' loan retention choices. We solve the model using backward induction. At $t = 2$, the price p depends on the measurement report r and also on any information about loan quality θ that the market can extract from observing the bank's retention choice α. The loan retention fraction α now acquires an informational role because the bank chooses α after observing θ. Specifically, to infer θ from α, suppose the market forms a conjecture $\alpha(\theta)$ about the bank's loan retention schedule. If the schedule $\alpha(\theta)$ is strictly monotone in θ (as it will turn out to be in equilibrium), the market can infer the exact value of θ. We denote the inferred value of θ as $\hat{\theta}(\alpha)$. The price p incorporates this inferred value $\hat{\theta}(\alpha)$ rather than the true value θ. Replacing θ with $\hat{\theta}(\alpha)$ in the pricing formula (8) yields

$$p^* = E[\widetilde{\pi}|\alpha, r, \hat{m}] - \delta(1-\alpha) = \beta r + (1-\beta)\left(\hat{\theta}(\alpha) + \hat{m}\right) - \delta(1-\alpha). \tag{16}$$

Next, we solve for the bank's choices of monitoring effort m and retention fraction α at $t = 1$. Substituting the transfer price (16) into the bank's payoff (6) and rearranging terms, we obtain

$$U(m,\alpha;\theta) = [\alpha + (1-\alpha)\beta](\theta + m) + (1-\alpha)(1-\beta)\left(\hat{\theta}(\alpha) + \hat{m}\right) \tag{17}$$
$$- \delta(1-\alpha)^2 - \frac{\tau}{2h_\eta}\left(\alpha^2 + (1-\alpha)^2\beta\right) - \frac{c}{2}m^2.$$

Differentiating (17) with respect to m yields $m^* = \frac{\alpha+(1-\alpha)\beta}{c}$. Substituting m^* into (17) and imposing the rational expectation requirement that $\hat{m} = m^*$ yields

$$U(m^*,\alpha;\theta) = [\alpha + (1-\alpha)\beta]\theta + (1-\alpha)(1-\beta)\hat{\theta}(\alpha) \tag{18}$$
$$+m^* - \frac{c}{2}(m^*)^2 - \delta(1-\alpha)^2 - \frac{\tau}{2h_\eta}\left(\alpha^2 + (1-\alpha)^2\beta\right).$$

Expression (18) suggests that the market inference $\hat{\theta}(\alpha)$ affects the bank's payoff and hence potentially changes the bank's equilibrium choice of loan retention $\alpha(\theta)$. Rational expectation equilibrium requires that the inference is consistent with the bank's equilibrium choice, i.e., $\hat{\theta}(\alpha(\theta)) = \theta$. As is standard in the literature (e.g., Spence, 1974), such a signaling equilibrium is sustained if the "single-crossing property" holds. In our environment, the single-crossing property requires that the bank with a higher loan quality (high θ) is willing to retain a higher fraction of its loan portfolio than the bank with a lower loan quality. To verify this property, note that the marginal rate of substitution between $\hat{\theta}$ and α in the bank's payoff (18) is

$$\frac{\partial\hat{\theta}}{\partial\alpha} = -\frac{U_\alpha}{U_{\hat{\theta}}} = -\frac{\theta - \hat{\theta}(\alpha)}{1-\alpha} - \frac{1-\beta}{c} + \frac{\tau(\alpha - (1-\alpha)\beta)}{(1-\alpha)(1-\beta)h_\eta} + \frac{2\delta(1-\alpha)}{(1-\alpha)(1-\beta)}, \tag{19}$$

which is strictly decreasing in the loan quality θ. In other words, a high θ bank is more willing to retain a higher proportion of the loan portfolio than a low θ bank for the same amount of improvement in the market inference. With the single-crossing property established, we now formally construct the fully revealing equilibrium in the following proposition.

Proposition 2 *Given the uniform measurement precision β,*

1. *the equilibrium loan retention schedule is given by*

$$\alpha_U(\theta;\beta) = 1 + \frac{c\tau}{\left((1-\beta)^2 + 2c\delta\right)h_\eta + (1+\beta)c\tau} w\left(-e^{-\left(1+\frac{(1-\beta)h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right), \qquad (20)$$

   where $w(.)$ is the Lambert W function (i.e., the principal solution for y in $x = ye^y$);

2. *the equilibrium loan retention schedule $\alpha_U(\theta;\beta)$ is strictly increasing in loan quality θ and the liquidity discount δ, but strictly decreasing in the degree of risk aversion τ, the monitoring cost c, and the residual variance of loan cash flows $\frac{1}{h_\eta}$.*

Proposition 2 is intuitive and states that a bank with a higher-quality loan portfolio is induced to transfer a smaller proportion of its portfolio in order to obtain a more favorable price.[12] Furthermore, due to its informational role, such a loan retention schedule exhibits excessive retention which is socially inefficient. More precisely, the following corollary shows that, absent measurement, banks retain a larger fraction of their loans when there is private information than the optimal fraction under no private information.

[12] Note that by setting $\delta = 0$, $c = \infty$ (so that $m = 0$) and $\beta = 0$, it is straightforward to verify that $\alpha_U(\theta;0)$ coincides with the fully revealing retention schedule in Leland and Pyle (1977).

Corollary 1 *Under no measurement ($\beta = 0$), banks retain larger fractions of loans when the loan quality θ is unobservable than the optimal fraction when θ is observable, i.e., $\alpha_U(\theta; 0) \geq \alpha_0$ for all θ where α_0 is defined in Proposition 1. The inequality is strict if $\theta > \underline{\theta}$.*

An implication of Corollary 1 is that, given the bank's over-retention incentives under no measurement, and that the loan retention schedule $\alpha_U(\theta; \beta)$ explicitly depends on β, the precision of the measurement rule, the optimal measurement rule should be fine-tuned in order to mitigate the over-retention inefficiency, i.e., shifting $\alpha_U(\theta; \beta)$ closer to α_0. Toward that end, the following proposition sheds light on how a uniform increase of measurement precision over no measurement affects the bank's over-retention incentives.

Proposition 3 *Consider a uniform marginal increase of measurement over no measurement (i.e., $\beta = 0$):*

1. *when $\frac{\tau}{2h_\eta} < \frac{1}{c}$, measurement shrinks over-retention for all banks, i.e.,*

$$\left.\frac{\partial \left(\alpha_U(\theta; \beta) - \alpha_0\right)}{\partial \beta}\right|_{\beta=0} < 0 \text{ for all } \theta > \underline{\theta}; \tag{21}$$

2. *but when $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, measurement exacerbates over-retention if the bank's equilibrium retention fraction is sufficiently small.*[13]

Proposition 3 suggests that measurement rules that mandate a *uniform* increase of measurement may not necessarily be beneficial. A sufficient condition under which uniform

[13]Note that since $\alpha_U(\theta; 0)$ is strictly increasing in θ, the condition that $\alpha_U(\theta; 0)$ is sufficiently small is equivalent to a condition that the loan quality θ is sufficiently small.

measurement diminishes over-retention and thus improves efficiency is that risk-sharing considerations are relatively weak compared to monitoring incentives (i.e., $\frac{\tau}{2h_\eta} < \frac{1}{c}$). But when risk-sharing considerations are significant, uniform measurement could actually worsen the over-retention inefficiency. Proposition 3 thus points to an efficiency gain from making asset transfer measurement rules *contingent* on observable bank characteristics. In particular, part 2 of Proposition 3 suggests that it is optimal to require less precise, or even no measurement if the bank has transferred most of its loans (i.e., when α is relatively small).

To provide some intuition for Proposition 3, it is instructive to investigate how increasing precision affects over-retention incentives. Recall that absent private information, the loan retention schedule α_0 is insensitive to the loan quality θ. But when there is private information, the loan retention schedule α_U is strictly increasing in θ. Therefore, over-retention becomes more severe if the retention schedule rises more steeply in the loan quality (i.e., $\frac{\partial \alpha_U}{\partial \theta}$ is large). To study the behavior of $\frac{\partial \alpha_U}{\partial \theta}$, we reproduce its expression (equation (45) in the Appendix) below, i.e.,

$$\frac{\partial \alpha_U}{\partial \theta} = \frac{\overbrace{(1-\alpha)(1-\beta)}^{\text{benefit of over-retention}}}{\underbrace{\frac{\tau\left[\alpha-(1-\alpha)\beta\right]}{h_\eta} - 2\delta(1-\alpha) - \frac{1-\alpha}{c}(1-\beta)^2}_{\text{loss of over-retention}}}. \tag{22}$$

Equation (22) illustrates how a uniform measurement rule affects the retention schedule. The numerator of (22) captures the benefit of over-retention stemming from improving the market inference $\hat{\theta}(\alpha)$ about the loan quality. This benefit increases as the weight on $\hat{\theta}(\alpha)$ in the bank's payoff (18) increases. Importantly, more precise measurement (i.e., a larger β) diminishes the weight placed on the inference, as the market relies more on the report and less on the level of retention in its pricing of the loans.[14] Stated differently, measurement reduces the bank's benefit of over-retention and shrinks the amount of excess retention in equilibrium. We call the latter effect, the *inference effect* of measurement.

The denominator of (22) represents the bank's net loss from over-retention. Recall from (14) that, absent information asymmetry, the bank sets the optimal loan retention amount by trading off the marginal gain from improving monitoring incentives against the marginal loss of risk sharing and the liquidity cost. Excess loan retention causes more risk-sharing loss relative to the monitoring gain, thus resulting in a net loss for the bank. Examining the denominator of (22) suggests that increasing measurement has ambiguous effects on the over-retention loss.[15] On the one hand, more precise measurement reduces the risk-sharing loss from excess retention due to the complementary roles of retention and measurement on risk-sharing. Over-retention, therefore, is less costly when the bank is already required to measure the loans transferred, compared with under no measurement. This *risk-sharing effect* of measurement, therefore, encourages the bank to retain more loans and exacerbates the over-retention inefficiency. On the other hand, improving measurement precision also decreases the benefits of loan retention on monitoring, because measurement is a substitute for loan retention in incentivizing the bank to monitor loans. This *monitoring effect* of measurement thus increases the net loss from over-retention, which curbs excess retention.

The overall effect of measurement on excess retention therefore depends on the interplay

[14] Mathematically, note that, from (18), the weight on $\hat{\theta}$ is exactly $(1-\alpha)(1-\beta)$ (i.e., the numerator of (22)) and strictly decreasing in the measurement precision β.

[15] Mathematically, the first two terms in the denominator of (22) represent the risk-sharing loss from excess retention and the liquidity cost, respectively, whereas the last term represents the monitoring gain. Note that the first and the last terms are decreasing in the measurement precision β. Hence the overall effect of β on the denominator of (22) is ambiguous.

among the risk-sharing effect that increases retention and the inference and the monitoring effects that decrease retention. When risk-sharing considerations are less important than monitoring considerations, the risk-sharing effect is dominated and a uniform increase of measurement always curbs over-retention. This explains the conditions in part 1 of Proposition 3. However, when risk-sharing considerations become more important, the risk-sharing effect can sometimes dominate, in which case measurement induces excessive retention, thereby impairing efficiency. The risk-sharing effect of measurement in inducing excessive retention becomes especially important when the bank has transferred a large proportion of its loan portfolio and hence more of its loans are subject to the price volatility stemming from measurement. This explains the conditions in part 2 of Proposition 3.

An implication of Proposition 3 is that the efficiency can be improved if one tailors asset transfer measurement rules to banks' characteristics such as their asset transfer decisions. We next solve for such an optimal contingent measurement rule.

3.2.2 Contingent measurement rule

We denote the optimal choice of the contingent measurement rule as $\beta(\alpha)$, which is a general function of the retention fraction α that will be determined in equilibrium. We solve the model using backward induction. Note first that given the measurement rule $\beta(\alpha)$, the loan transfer price p and the bank's monitoring effort m^* are the same as those derived under the uniform rule, i.e., equations (16) and (10), respectively.

Next, we derive the bank's equilibrium loan retention schedule $\alpha(\theta)$. Such schedule $\alpha(\theta)$ must satisfy a bank's incentive compatibility (IC) constraints, i.e., a type θ bank must prefer choosing $\alpha(\theta)$ over the retention choice $\alpha(\theta')$ designed for type $\theta' \neq \theta$. Without loss of generality, let's assume that $\theta' > \theta$. Importantly, note that the optimal design of the contingent rule $\beta(\alpha)$ affects the IC constraints and, through this channel, influences the bank's retention schedule. To illustrate the effect of the contingent measurement rule, we now formally derive the bank's IC constraints. If the type θ bank chooses $\alpha(\theta)$, substituting $\beta = \beta(\alpha)$ and $\alpha = \alpha(\theta)$ into (18) yields the payoffs

$$
\begin{aligned}
U(\theta) &\equiv U(m^*(\alpha(\theta)), \alpha(\theta); \theta) \\
&= \theta + m^*(\alpha(\theta)) - \frac{c}{2}(m^*(\alpha(\theta)))^2 - \frac{\tau}{2h_\eta}\left[\alpha(\theta)^2 + (1-\alpha(\theta))^2\beta(\alpha(\theta))\right] - \delta(1-\alpha(\theta))^2,
\end{aligned} \tag{23}
$$

where the monitoring effort $m^*(\alpha(\theta)) = \frac{\alpha(\theta)+(1-\alpha(\theta))\beta(\alpha(\theta))}{c}$. Note that we have imposed the rational expectation requirement that the market inference is consistent with the bank's equilibrium choice, i.e., $\hat{\theta}(\alpha(\theta)) = \theta$. But if the type θ bank deviates from the equilibrium schedule and instead chooses $\alpha(\theta')$, its payoffs equal

$$
\begin{aligned}
U(\theta', \theta) &\equiv U(m^*(\alpha(\theta')), \alpha(\theta'); \theta) \\
&= [\alpha(\theta') + (1-\alpha(\theta'))\beta(\alpha(\theta'))]\theta + (1-\alpha(\theta'))(1-\beta(\alpha(\theta')))\theta' + m^*(\alpha(\theta')) \\
&\quad - \frac{c}{2}(m^*(\alpha(\theta')))^2 - \frac{\tau}{2h_\eta}\left[\alpha(\theta')^2 + (1-\alpha(\theta'))^2\beta(\alpha(\theta'))\right] - \delta(1-\alpha(\theta'))^2.
\end{aligned} \tag{24}
$$

Note that deviation of type θ to type $\theta^{'}$ by choosing $\alpha(\theta')$ instead of $\alpha(\theta)$ results in two differences between $U(\theta)$ and $U(\theta', \theta)$. First, the market inference about the bank's loan quality changes as they infer the bank's type to be θ' upon observing a retention amount of $\alpha(\theta')$. Second, under the contingent rule, the bank that changes its asset transfer decision is required to adopt a different precision level $\beta(\alpha(\theta'))$ regarding how the bank measures the transfer. This feature implies that one may influence the bank's asset transfer decision by imposing different measurement requirements for any type $\theta' \neq \theta$.

To ensure no deviation by the type θ bank, IC constraints require that $U(\theta) \geq U(\theta', \theta)$. Substituting (23) and (24) into the IC constraint and rearranging terms yields

$$
\begin{aligned}
&U(\theta) \geq U(\theta'; \theta) \\
&\quad = [\alpha(\theta') + (1-\alpha(\theta'))\beta(\alpha(\theta'))](\theta - \theta') \\
&\quad + [\alpha(\theta') + (1-\alpha(\theta'))\beta(\alpha(\theta'))]\theta' + (1-\alpha(\theta'))(1-\beta(\alpha(\theta')))\theta' \\
&\quad + m^*(\alpha(\theta')) - \frac{c}{2}(m^*(\alpha(\theta')))^2 - \frac{\tau}{2h_\eta}\left[\alpha(\theta')^2 + (1-\alpha(\theta'))^2\beta(\alpha(\theta'))\right] - \delta(1-\alpha(\theta'))^2 \\
&\quad = U(\theta') - \underbrace{[\alpha(\theta') + (1-\alpha(\theta'))\beta(\alpha(\theta'))](\theta' - \theta)}_{\text{deviation loss} > 0}. \qquad (25)
\end{aligned}
$$

The IC constraint (25) implies the payoffs $U(\theta'; \theta)$ of type θ who chooses the retention allocation of a higher type θ' is strictly lower than $U(\theta)$, the payoffs of type θ by

$$
[\alpha(\theta') + (1-\alpha(\theta'))\beta(\alpha(\theta'))](\theta' - \theta), \qquad (26)
$$

which captures the expected loss from deviation. Intuitively, to prevent the type θ bank from choosing the retention of the higher type θ', the type θ' bank is induced to increase its retention level $\alpha(\theta')$, which reduces $U(\theta'; \theta)$ in equilibrium. Furthermore, the deviation loss term also suggests that requiring a higher measurement precision can reduce the marginal effect of excess loan retention in deterring the type θ bank from mimicking type θ'. In fact, if measurement is perfect (i.e., $\beta(\alpha(\theta')) = 1$), increasing $\alpha(\theta')$ would have no impact on the deviation term. This suggests a disciplinary role of measurement in curbing over-retention. Intuitively, more precise measurement makes the loan transfer price depend more on the measurement report (which reflects the bank's *true* loan quality) and less on the market's prior expectation about the loan cash flow (which is formed based on the *inferred* loan quality), thus weakening the bank's incentive to influence the market inference via loan retention.[16] Accordingly, an implication from examining the IC constraint (25) is that, when the loan retention is overly high, the measurement precision should be increased in order to curb such over-retention. We will verify that this implication is indeed true when we solve for the optimal measurement rule.

Next, to ensure no deviation by the type θ' bank, the analogous IC constraint requires that $U(\theta') \geq U(\theta;\theta')$, which is given by

$$U(\theta') \geq U(\theta) + [\alpha(\theta) + (1-\alpha(\theta))\,\beta(\alpha(\theta))]\,(\theta' - \theta)\,. \tag{27}$$

Combining (25) and (27) yields

$$[\alpha(\theta) + (1-\alpha(\theta))\,\beta(\alpha(\theta))]\,(\theta'-\theta) \leq U(\theta') - U(\theta) \leq [\alpha(\theta') + (1-\alpha(\theta'))\,\beta(\alpha(\theta'))]\,(\theta'-\theta)\,. \tag{28}$$

Note that (28) implies that for $\theta' > \theta$, $\alpha(\theta') + (1-\alpha(\theta'))\,\beta(\alpha(\theta')) > \alpha(\theta) + (1-\alpha(\theta))\,\beta(\alpha(\theta))$, i.e., $\alpha(\theta) + (1-\alpha(\theta))\,\beta(\alpha(\theta))$ is non-decreasing in θ. Taking the limit of $\theta' \to \theta$, we obtain the IC constraint as:

$$U'(\theta) = \alpha(\theta) + (1-\alpha(\theta))\,\beta(\alpha(\theta)). \tag{29}$$

[16]The *ex-post* disciplining role of measurement in alleviating over-retention is similar to the disciplinary role of a performance report first developed by Kanodia and Lee (1998).

Formally, we next state the conditions under which the retention schedule $\alpha(\theta)$ is incentive compatible.

Lemma 2 *Given the measurement rule $\beta(\alpha)$, the loan retention schedule $\alpha(\theta)$ is incentive compatible if and only if:*

1. $U'(\theta) = \alpha(\theta) + (1-\alpha(\theta))\,\beta(\alpha(\theta))$ *and*

2. $\alpha(\theta) + (1-\alpha(\theta))\,\beta(\alpha(\theta))$ *is non-decreasing in* θ.

As is standard in the adverse selection literature, incentive compatibility requires both (29) and a monotonicity condition. In deriving the optimal schedules of $\alpha(\theta)$ and $\beta(\alpha)$, we next ignore the monotonicity condition and verify that the optimal schedules from the relaxed problem do indeed satisfy the monotonicity condition. Therefore, the optimal contingent measurement rule $\beta(\alpha)$ is the solution to the following optimization program

$$\max_{\beta(\alpha)} W \equiv \int_{\underline{\theta}}^{\overline{\theta}} U(\theta) f(\theta) d\theta, \tag{30}$$

$$s.t.\ U'(\theta) = \alpha(\theta) + (1-\alpha(\theta))\,\beta(\alpha(\theta)).$$

We solve program (30) by solving the following optimal control problem

$$\max_{\alpha(\theta),\beta(\theta)} W \equiv \int_{\underline{\theta}}^{\overline{\theta}} U(\theta) f(\theta) d\theta, \tag{31}$$

$$s.t.\ U'(\theta) = \alpha(\theta) + (1-\alpha(\theta))\,\beta(\theta). \tag{32}$$

We establish in the Appendix that programs (30) and (31) are equivalent. In other words, to derive the optimal contingent measurement rule, we can first solve for the optimal schedules of measurement precision and loan retention as a function of the bank's loan quality θ, i.e., $\{\alpha^*(\theta), \beta^*(\theta)\}$, although θ is not directly observable and hence the measurement rule cannot be made contingent on θ. The optimal contingent rule $\beta_C(\alpha)$ is then given by inverting the optimal schedule of loan retention and substituting it into the optimal measurement schedule, i.e., $\beta_C(\alpha) = \beta^*(\theta^*(\alpha))$, where $\theta^*(\alpha)$ is the inverse function of $\alpha^*(\theta)$. Solving program (31) yields the optimal loan retention and measurement rule schedules that we formally state next.

Proposition 4 *When loan quality θ is unobservable, the optimal loan retention and measurement schedules, $\{\alpha^*(\theta), \beta_C(\alpha)\}$ satisfy:*

1. *the optimal measurement schedule is a contingent rule that sets precision $\beta_C(\alpha) = \frac{3\alpha-1}{1-\alpha} - 4\delta\frac{h_\eta}{\tau}$ if $\alpha > \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$ and requires no measurement if $\alpha \leq \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$;*

2. *if $\frac{\tau}{2h_\eta} < \frac{1}{c}$, the equilibrium loan retention schedule $\alpha^*(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$ satisfies*

$$\frac{\partial\alpha^*(\theta)}{\partial\theta} = H(\alpha^*(\theta)), \tag{33}$$

and the equilibrium measurement precision $\beta_C(\alpha^*(\theta)) > 0$, where the function

$$H(x) \equiv \frac{c\left(1 + 2\delta\frac{h_\eta}{\tau} - 2x\left(1 + \delta\frac{h_\eta}{\tau}\right)\right)}{c(1-x)\left(\frac{\tau}{h_\eta} + \delta\right) - 4\left(1 + \delta\frac{h_\eta}{\tau}\right)\left(1 + 2\delta\frac{h_\eta}{\tau} - 2x\left(1 + \delta\frac{h_\eta}{\tau}\right)\right)};$$

3. *but if $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, there exists a measurement cutoff $\theta_c \in [\underline{\theta}, \overline{\theta}]$, where θ_c solves $\alpha_U(\theta_c; 0) = \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$ such that, for $\theta > \theta_c$, $\alpha^*(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$ solves (33) and $\beta_C(\alpha^*(\theta)) > 0$, while for $\theta \leq \theta_c$, $\alpha^*(\theta) = \alpha_U(\theta; 0) \in (0, \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}]$ and $\beta_C(\alpha^*(\theta)) = 0$, i.e., no*

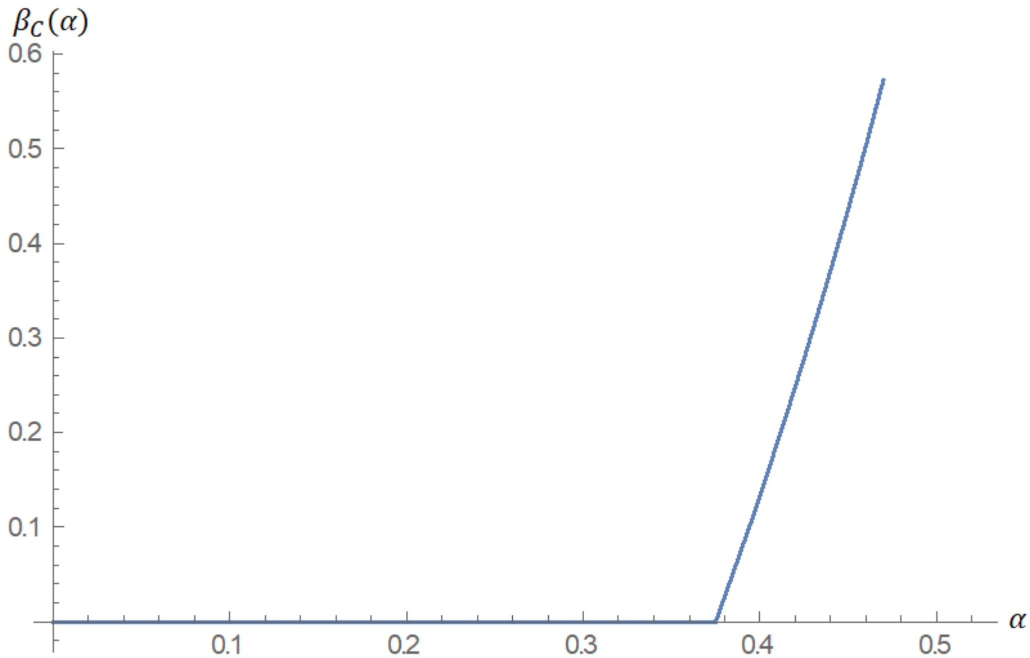

Figure 2: Optimal contingent measurement precision $\beta_C(\alpha)$ as a function of loan retention fraction α. The following parameter values are used in this plot: $\delta = 0.1$, $c = 2$, $\tau = 2$ and $h_\eta = 1$.

measurement should be required.[17]

Proposition 4 states that, when banks have private information about the quality of their loan portfolios, the optimal measurement rule for loan transfers should be made contingent on the amount of loans transferred. In particular, banks should measure their loan portfolios if and only if they retain a sufficiently large proportion of those loans *and* the precision of such measurement should increase in the proportion of loan retention. Figure 2 provides a graphic illustration of the optimal contingent measurement rule. The intuition for Proposition 4 follows from the impact of measurement on the over-retention inefficiency discussed in Proposition 3. Recall that measurement can worsen the over-retention inefficiency if the bank has transferred most of its loans (i.e., whenever α is small). Accordingly, the optimal

[17]Recall that the expression of $\alpha_U(\theta; 0)$ is as given in Proposition 2.

rule should require no measurement under those circumstances.

We now relate the implications of Proposition 4 to current accounting standards on asset transfers. As discussed previously, the central issue at debate is whether and when transferred assets should be recognized on the balance sheet (i.e., treated as collateralized borrowing) or derecognized (i.e., treated as a sale). Under current standards, the key guiding principle for derecognition is whether the transferor has surrendered control. To the extent that derecognition implies less measurement or even no measurement of the performance of the loans, the contingent measurement rule derived in Proposition 4 provides conditions under which derecognition is desirable. In this light, Proposition 4 states that no measurement should be allowed when the bank has transferred most of the loans. Moreover, to the extent that the degree of control is negatively associated with the fraction of assets transferred, Proposition 4 lends some support for adopting the control principle in the current accounting standards.

Furthermore, because the contingent measurement rule depends on the bank's asset transfer decision, the exogenous parameters that drive the bank's equilibrium choice of asset transfer provides some insights into environments when measurement is more likely to be useful. In particular, when monitoring considerations are more important relative to risk-sharing considerations (i.e., $\frac{\tau}{2h_\eta} < \frac{1}{c}$), measurement always occurs in order to provide efficient monitoring incentives. But when risk-sharing considerations become sufficiently more important (i.e., $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$), Proposition 4 suggests that there exists a measurement cutoff θ_c on the bank's loan quality below which measurement never occurs.

Our contingent measurement rule implies that the one-size-fits-all risk retention requirement of the Dodd-Frank Act may be suboptimal. The Dodd-Frank Act requires securitization sponsors to retain no less than a 5% share of the aggregate credit risk of the assets they securitize. But as discussed above, optimal risk retention level depend on the exogenous parameters that determine a bank's monitoring considerations vs. risk sharing considerations and those parameters are likely to vary across banks. More importantly, the optimal risk retention level depends on a bank's information environment that is also likely to vary across banks.

Interestingly, the equilibrium measurement schedule $\beta_C(\alpha)$ resembles the threshold disclosure strategy derived in the voluntary disclosure literature in the sense that disclosure occurs only upon good news (i.e., when the loan quality θ is sufficiently good); yet, the mechanisms under which the two equilibria are sustained are completely different. In our model, the measurement cutoff arises due to the optimal design of the *ex-ante* mandatory measurement rule, whereas, in the voluntary disclosure literature, the disclosure threshold prevails as a consequence of firms' own *ex-post* voluntary disclosure choice. To generate additional implications, we next provide some comparative statics on the equilibrium retention fraction $\alpha^*(\theta)$, the equilibrium measurement cutoff θ_c, and the equilibrium measurement precision $\beta^*(\theta) \equiv \beta_C(\alpha^*(\theta))$ when asset transfers are measured.

Corollary 2 *The comparative statics of the equilibrium retention fraction $\alpha^*(\theta)$, the equilibrium measurement precision $\beta^*(\theta)$, and the equilibrium measurement cutoff θ_c are as follows:*

1. *both $\alpha^*(\theta)$ and $\beta^*(\theta)$ are strictly increasing in the loan quality θ and the liquidity discount δ, and strictly decreasing in the degree of risk aversion τ, the monitoring cost c, and the residual variance of loan cash flows $\frac{1}{h_\eta}$;*

2. *if $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, θ_c is strictly decreasing in the liquidity discount δ, and strictly increasing in*

the degree of risk aversion τ*, the monitoring cost* c*, and the residual variance of loan cash flows* $\frac{1}{h_\eta}$.

Corollary 2 is mostly intuitive given the preceding discussion. Under the optimal measurement rule, measurement is less likely to occur and the measurement precision is lower when risk-sharing considerations overwhelm monitoring considerations–when the bank is more risk averse, loan cash flows are more volatile, and/or it is more costly to induce monitoring.

However, Corollary 2 also carries a key implication regarding how the (il)liquidity of the secondary loan sale market δ should affect the benefit of measurement. To better appreciate this implication, recall that, absent the private information about loan quality θ, the optimal measurement precision β_0 is independent of the liquidity effect δ (Proposition 1). In contrast, Corollary 2 suggests that when there is private information and the liquidity of the loan sale market deteriorates (i.e., δ increases), more measurement is warranted in the sense that, under the optimal measurement rule, measurement is more likely to occur and upon measurement, the precision is higher. The intuition for this result is as follows. In a less liquid market, the bank has weaker incentives to transfer loans due to the liquidity cost and hence chooses to retain more loans. Furthermore, the optimal measurement rule requires that more measurement should be required when the bank retains more loans. Stated differently, that lower market liquidity calls for more measurement and greater transparency is a direct consequence of the optimal contingent measurement rule.

Proposition 4 also sheds light on how the contingent measurement rule affects the bank's loan retention schedule. It suggests that, interestingly, the measurement rule results in a "kink" point in the loan retention schedule. When the loan quality θ is lower than the

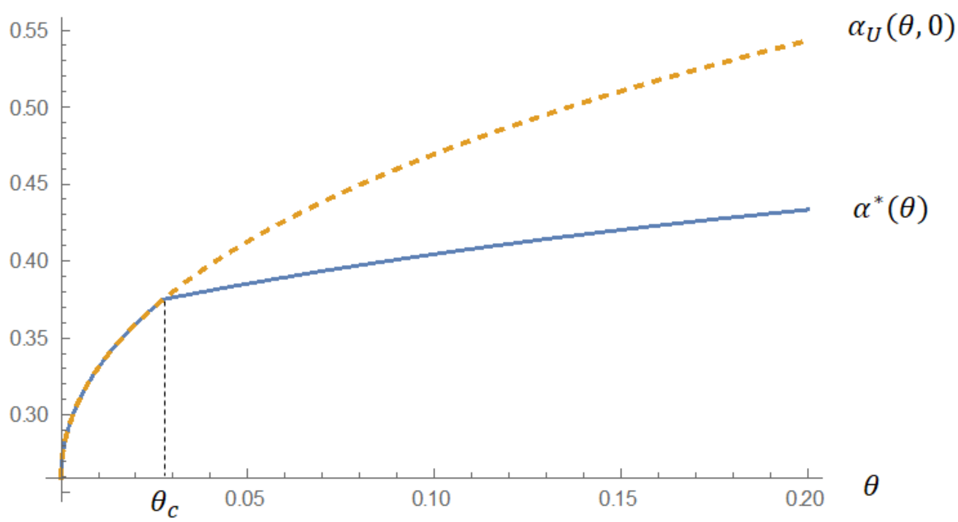

Figure 3: Equilibrium loan retention schedule $\alpha^*(\theta)$ under optimal measurement rule as a function of loan quality θ. The following parameter values are used in this plot: $\delta = 0.1$, $c = 2$, $\tau = 2$ and $h_\eta = 1$.

measurement cutoff θ_c, the loan retention schedule under the contingent measurement rule overlaps with that under no-measurement. However, the loan retention schedule becomes less steep and falls below the schedule under no-measurement, as the retention amount passes the cutoff that triggers measurement (i.e., $\theta > \theta_c$). We summarize this result in the following corollary.

Corollary 3 *The comparison between the loan retention fraction under the optimal contingent measurement rule $\alpha^*(\theta)$ and that under no-measurement $\alpha_U(\theta; 0)$ is as follows:*

1. *if $\frac{\tau}{2h_\eta} < \frac{1}{c}$, $\alpha^*(\theta) < \alpha_U(\theta; 0)$ for all θ;*

2. *if $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, $\alpha^*(\theta) = \alpha_U(\theta; 0)$ if $\theta \leq \theta_c$, whereas $\alpha^*(\theta) < \alpha_U(\theta; 0)$ if $\theta > \theta_c$.*

Figure 3 provides a graphic illustration of Corollary 3 in the case of $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. Intuitively, when the loan quality is unobservable, the bank over-retains loans and retains even more when the loans are of higher quality. As the loan quality improves above some level, the bank retains a sufficient amount of loans that triggers measurement. The measurement, in turn, curbs the bank's over-retention motive, shifts the retention amount downward, and thus results in a kink in the equilibrium loan retention schedule.

Finally, Proposition 4 generates insights into the real effects of asset transfer measurement rule and describes how the measurement rule affects the bank's monitoring incentives, which determines the performance of the bank's loan portfolio. We find that the optimal contingent measurement rule results in higher monitoring effort and thus improves the loan performance, relative to no measurement. We summarize this result in the following corollary.

Corollary 4 *The comparison between the monitoring effort under the optimal contingent measurement rule* $m(\theta) \equiv m^*(\alpha^*(\theta))$ *and that under no-measurement* $m_U(\theta) \equiv m^*(\alpha_U(\theta; 0))$ *is as follows:*

1. *if* $\frac{\tau}{2h_\eta} < \frac{1}{c}$, $m(\theta) > m_U(\theta)$ *for all* θ;

2. *if* $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, $m(\theta) = m_U(\theta)$ *if* $\theta \leq \theta_c$, *whereas* $m(\theta) > m_U(\theta)$ *if* $\theta > \theta_c$.

Figure 4 provides a graphic illustration of Corollary 4 in the case of $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. It suggests that the optimal measurement rule plays a beneficial role in mitigating the problem of banks' reduced monitoring incentives after loan transfers. As previously explained, measurement substitutes for loan retention in incentivizing the bank to monitor, which helps to maintain an adequate level of monitoring effort even after the bank off-loads some of its loans.

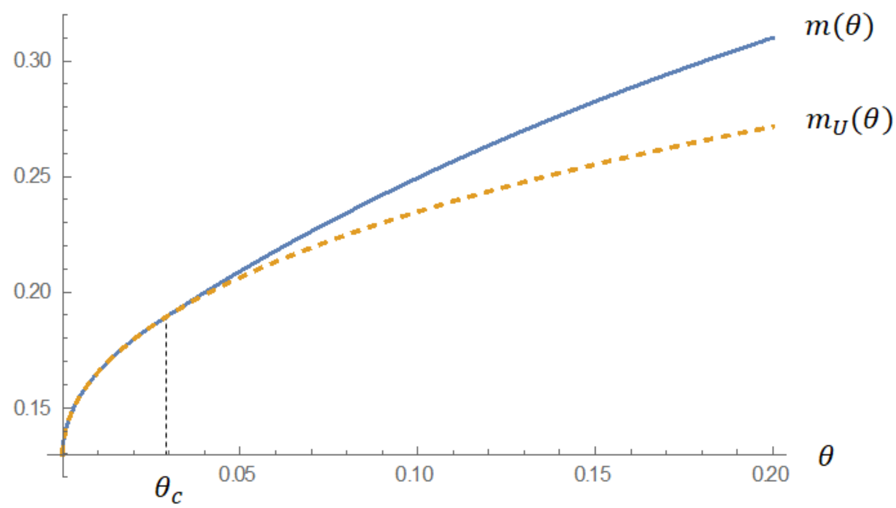

Figure 4: Equilibrium monitoring effort under optimal measurement rule, $m(\theta) \equiv m^*(\alpha^*(\theta))$, as a function of loan quality θ. The following parameter values are used in this plot: $\delta = 0.1$, $c = 2$, $\tau = 2$ and $h_\eta = 1$.

4 Conclusion

We develop a model of a representative bank to study the trade-offs that banks face in engaging in asset transfers. Given those trade-offs, we study how *ex-ante* measurement rules affect asset transfer policies. Our main result is that, in the presence of monitoring and informational frictions, a contingent measurement rule is optimal: banks should report the performance of transferred loans if and only if the amount of loans retained is sufficiently high.

To focus exclusively on measurement rules, we study the simplest form of credit risk transfer in which banks sell their loans proportionally without recourse in a secondary market. To the extent that the degree of control is negatively associated with the fraction of assets transferred, our contingent measurement rule lends some support for adopting the control principle in determining the appropriate accounting treatment for transfers. We believe that to model control issues, one needs to incorporate richer institutional features of asset transfers such as securitization that our model does not capture. Future research may expand our framework of asset transfer and measurement to incorporate such features and examine how they interact with measurement rules.

Finally, we do not model regulatory capital that plays an important role in affecting banks' incentives to engage in asset transfers. As we have shown in prior work, accounting measurements play a crucial role in the design of regulatory capital (Mahieux, Sapra, and Zhang, 2021). It would therefore be useful to investigate how such measurements interact with regulatory capital to affect loan transfer decisions. We leave this important and interesting issue to future research.

References

[1] Allen, Franklin, and Elena Carletti. "Credit risk transfer and contagion." Journal of Monetary Economics 53, no. 1 (2006): 89-111.

[2] Barth, Mary E., Gaizka Ormazabal, and Daniel J. Taylor. "Asset securitizations and credit risk." The Accounting Review 87, no. 2 (2012): 423-448.

[3] Bertomeu, Jeremy, Lucas Mahieux, and Haresh Sapra. "Interplay between accounting and prudential regulation." Available at SSRN (2020).

[4] Bleck, Alexander, and Pingyang Gao. "Mark-to-market, loan retention, and loan origination." Chicago Booth Research Paper (2017): 10-06.

[5] Corona, Carlos, Lin Nan, and Gaoqing Zhang. "Accounting Information Quality, Interbank Competition, and Bank Risk Taking." The Accounting Review 90 (2015): 967-985.

[6] Corona, Carlos, Lin Nan, and Gaoqing Zhang. "Banks' asset reporting frequency and capital regulation: An analysis of discretionary use of fair-value accounting." The Accounting Review 94, no. 2 (2019): 157-178.

[7] Corona, Carlos, Lin Nan, and Gaoqing Zhang. "The Coordination Role of Stress Tests in Bank Risk Taking." Journal of Accounting Research 57, no. 5 (2019): 1161-1200.

[8] Dahiya, Sandeep, Manju Puri, and Anthony Saunders. "Bank borrowers and loan sales: New evidence on the uniqueness of bank loans." The Journal of business 76, no. 4 (2003): 563-582.

[9] DeMarzo, Peter, and Darrell Duffie. "A liquidity-based model of security design." Econometrica 67, no. 1 (1999): 65-99.

[10] Dou, Yiwei, Stephen G. Ryan, and Biqin Xie. "The real effects of FAS 166/167 on banks' mortgage approval and sale decisions." Journal of Accounting Research 56, no. 3 (2018): 843-882.

[11] Dechow, Patricia M., and Catherine Shakespeare. "Do managers time securitization transactions to obtain accounting benefits?" The Accounting Review 84, no. 1 (2009): 99-132.

[12] Financial Accounting Standards Board news release, available at https://www.fasb.org/jsp/FASB/FASBContent_C/NewsPage&cid=1176156240834, June 2009.

[13] Financial Accounting Standards Board. Transfers and Servicing (Topic 860). No. 2014-11, June 2014.

[14] Gao, Pingyang, and Xu Jiang. "Reporting choices in the shadow of bank runs." Journal of Accounting and Economics 65, no. 1 (2018): 85-108.

[15] Goldstein, Itay, and Yaron Leitner. "Stress tests and information disclosure." Journal of Economic Theory 177 (2018): 34-69.

[16] Gorton, Gary B., and George G. Pennacchi. "Banks and loan sales marketing nonmarketable assets." Journal of Monetary Economics 35, no. 3 (1995): 389-411.

[17] Greenbaum, Stuart I., and Anjan V. Thakor. "Bank funding modes: Securitization versus deposits." Journal of Banking and Finance 11, no. 3 (1987): 379-401.

[18] Greenbaum, Stuart I., Anjan V. Thakor, and Arnoud Boot. Contemporary financial intermediation. Academic Press, 2019.

[19] Hirshleifer, Jack. "The private and social value of information and the reward to incentive

activity." American Economic Review 61 (1971), 561-574.

[20] Kanodia, Chandra, and Deokheon Lee. "Investment and disclosure: The disciplinary role of periodic performance reports." Journal of Accounting Research 36, no. 1 (1998): 33-55.

[21] Keys, Benjamin J., Tanmoy Mukherjee, Amit Seru, and Vikrant Vig. "Did securitization lead to lax screening? Evidence from subprime loans." The Quarterly Journal of Economics 125, no. 1 (2010): 307-362.

[22] Leland, Hayne E., and David H. Pyle. "Informational asymmetries, financial structure, and financial intermediation." The Journal of Finance 32, no. 2 (1977): 371-387.

[23] Liang, Pierre Jinghong and Gaoqing Zhang. "On the Social Value of Accounting Objectivity in Financial Stability." The Accounting Review 94, no 1 (2019): 229-248.

[24] Lu, Tong, Haresh Sapra, and Ajay Subramanian. "Agency Conflicts, Bank Capital Regulation, and Marking-to-Market." The Accounting Review 94, no 6 (2019): 365-384.

[25] Mahieux, Lucas. "Fair Value Accounting, Illiquid Assets, and Financial Stability." Working paper, September, 2021.

[26] Mahieux, Lucas, Haresh Sapra, and Gaoqing Zhang. "CECL: Timely Loan Loss Provisioning and Banking Regulation." (April 29, 2020). Available at SSRN: https://ssrn.com/abstract=3523321.

[27] Marsh, Ian W. "The effect of lenders' credit risk transfer activities on borrowing firms' equity returns." Cass Business School Research Paper, Bank of Finland Research Discussion Paper 31 (2006).

[28] Parlour, Christine A., and Guillaume Plantin. "Loan sales and relationship banking." The

Journal of Finance 63, no. 3 (2008): 1291-1314.

[29] Pennacchi, George G. “Loan sales and the cost of bank capital.” The Journal of Finance 43, no. 2 (1988): 375-396.

[30] Plantin, Guillaume, Haresh Sapra, and Hyun Song Shin. “Marking-to-market: panacea or Pandora’s box?.” Journal of Accounting Research 46, no. 2 (2008): 435-460.

[31] Pozsar, Zoltan, Tobias Adrian, Adam Ashcraft, and Hayley Boesky. “Shadow banking.” New York Fed staff report no. 458 (2010).

[32] Spence, Michael. “Competitive and optimal responses to signals: An analysis of efficiency and distribution.” Journal of Economic Theory 7, 3 (1974): 296-332.

[33] Stein, Jeremy C. “Securitization, shadow banking, and financial fragility.” Daedalus 139, no. 4 (2010): 41-51.

[34] Zhang, Gaoqing. “Competition and Opacity in the Financial System.” Forthcoming, Management Science (2019).

Appendix: proofs

Proof. of Lemma 1: See the main text. ■

Proof. of Proposition 1: Substituting the equilibrium monitoring effort in (10) into (13) and (14) yields:

$$\frac{1-\beta}{c} = \frac{\tau}{2h_\eta}, \tag{34}$$

$$\frac{(1-\alpha)(1-\beta)^2}{c} + 2\delta(1-\alpha) = \frac{\tau\left[\alpha-(1-\alpha)\beta\right]}{h_\eta}. \tag{35}$$

If $\frac{\tau}{2h_\eta} < \frac{1}{c}$, (34) gives $\beta_0 = 1 - \frac{c\tau}{2h_\eta} > 0$. Solving (35) gives:

$$\alpha_0(\beta) = \frac{((1-\beta)^2 + 2c\delta)h_\eta + \beta c\tau}{((1-\beta)^2 + 2c\delta)h_\eta + (1+\beta)c\tau}. \tag{36}$$

Substituting β_0 into (36) gives $\alpha_0 = \frac{8\delta h_\eta^2 + 4h_\eta\tau - c\tau^2}{8\delta h_\eta^2 + \tau(8h_\eta - c\tau)}$. If $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, the left-hand side of (34) is always smaller than the right-hand side, i.e.,

$$\frac{1-\beta}{c} \leq \frac{1}{c} \leq \frac{\tau}{2h_\eta}.$$

Therefore, $\beta_0 = 0$. Substituting $\beta_0 = 0$ into (36) gives $\alpha_0 = \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}$. ■

Proof. of Proposition 2: In equilibrium, since $\hat{\theta}(\alpha) = \theta$, replacing $\hat{\theta}$ with θ in (18) yields the bank's equilibrium payoff for a given θ:

$$
\begin{aligned}
U(\theta) &\equiv U\left(m^*\left(\alpha\left(\theta\right)\right), \alpha\left(\theta\right); \theta\right) \\
&= \theta + \frac{\alpha\left(\theta\right) + \left(1-\alpha\left(\theta\right)\right)\beta}{c} - \frac{\left[\alpha\left(\theta\right) + \left(1-\alpha\left(\theta\right)\right)\beta\right]^2}{2c} \\
&- \frac{\tau}{2}\left(\frac{\alpha\left(\theta\right)^2 + \left(1-\alpha\left(\theta\right)\right)^2\beta}{h_\eta}\right) - \delta\left(1-\alpha\left(\theta\right)\right)^2.
\end{aligned}
\tag{37}
$$

Consider a deviation in which the bank chooses a different retention fraction $\alpha\left(\theta'\right)$ rather than $\alpha\left(\theta\right)$. Without loss of generality, let $\theta' > \theta$. The bank's payoff is then given by:

$$
\begin{aligned}
U\left(\theta'; \theta\right) &\equiv U\left(m^*\left(\alpha\left(\theta'\right)\right), \alpha\left(\theta'\right); \theta\right) \qquad (38) \\
&= \left[\alpha\left(\theta'\right) + \left(1-\alpha\left(\theta'\right)\right)\beta\right]\theta + \left(1-\alpha\left(\theta'\right)\right)\left(1-\beta\right)\theta' \\
&+ \left(\alpha(\theta') + \left(1-\alpha(\theta')\right)\right)\frac{\alpha\left(\theta'\right) + \left(1-\alpha\left(\theta'\right)\right)\beta}{c} - \frac{\left[\alpha\left(\theta'\right) + \left(1-\alpha\left(\theta'\right)\right)\beta\right]^2}{2c} \\
&- \frac{\tau}{2}\left(\frac{\alpha\left(\theta'\right)^2 + \left(1-\alpha\left(\theta'\right)\right)^2\beta}{h_\eta}\right) - \delta\left(1-\alpha\left(\theta'\right)\right)^2.
\end{aligned}
$$

The incentive-compatible (IC) constraint requires that

$$
U\left(\theta\right) \geq U\left(\theta'; \theta\right), \tag{39}
$$

which can be simplified into

$$
U\left(\theta'\right) - U\left(\theta\right) \leq \left[\alpha\left(\theta'\right) + \left(1-\alpha\left(\theta'\right)\right)\beta\right]\left(\theta' - \theta\right). \tag{40}
$$

Analogously, the IC for the bank with θ' requires that

$$U(\theta') \geq U(\theta; \theta'), \tag{41}$$

which can be simplified into

$$U(\theta') - U(\theta) \geq [\alpha(\theta) + (1 - \alpha(\theta))\beta](\theta' - \theta). \tag{42}$$

Combining (40) and (42) yields:

$$[\alpha(\theta) + (1 - \alpha(\theta))\beta](\theta' - \theta) \leq U(\theta') - U(\theta) \leq [\alpha(\theta') + (1 - \alpha(\theta'))\beta](\theta' - \theta). \tag{43}$$

Taking the limit of $\theta' \to \theta$ gives:

$$U'(\theta) = \alpha(\theta) + (1 - \alpha(\theta))\beta. \tag{44}$$

As similarly shown in Lemma 2, (44) and a monotonicity condition that $\alpha(\theta)$ is strictly increasing in θ are sufficient and necessary for the IC constraints. We next ignore the monotonicity condition and derive the optimal retention schedule $\alpha(\theta)$. Later we verify the equilibrium $\alpha(\theta)$ indeed satisfies the monotonicity condition.

Differentiating $U(\theta)$ in (37) with respect to θ and substituting into (44) gives:

$$\frac{\partial \alpha(\theta)}{\partial \theta} = \frac{(\alpha(\theta) - 1)(1 - \beta)}{\frac{1-\alpha(\theta)}{c}(1 - \beta)^2 + 2\delta(1 - \alpha(\theta)) - \frac{\tau(\alpha(\theta) - (1-\alpha(\theta))\beta)}{h_\eta}}. \tag{45}$$

The initial condition for (45) is given by type $\underline{\theta}$ bank's choice. Type $\underline{\theta}$ bank chooses α to maximize $U(\underline{\theta})$ in (37) and taking the first-order condition gives:

$$\alpha_U(\underline{\theta};\beta) = \frac{((1-\beta)^2 + 2c\delta)h_\eta + \beta c\tau}{((1-\beta)^2 + 2c\delta)h_\eta + (1+\beta)c\tau}. \tag{46}$$

Solving the differential equation yields

$$\alpha_U(\theta;\beta) = 1 + \frac{c\tau}{((1-\beta)^2 + 2c\delta)h_\eta + (1+\beta)c\tau} w\left[-e^{-\left(1+\frac{(1-\beta)h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right], \tag{47}$$

where $w(.)$ is the Lambert W function (i.e., the principal solution for y in $x = ye^y$). This proves part 1 of the proposition.

Note that, by observing the expression of $\alpha_U(\theta;\beta)$, it is strictly increasing in θ. This also verifies the monotonicity requirement of the IC constraints. In addition, recall that $\alpha_U(\theta,\beta)$ is defined such that

$$\frac{\partial \alpha_U(\theta,\beta)}{\partial \theta} = \frac{(1-\alpha_U(\theta,\beta))(1-\beta)}{\frac{\tau}{h_\eta}\left(\alpha_U(\theta,\beta) - (1-\alpha_U(\theta,0))\beta\right) - 2\delta(1-\alpha_U(\theta,\beta)) - \frac{1}{c}(1-\alpha_U(\theta,\beta))(1-\beta)^2}. \tag{48}$$

It is straightforward to verify that, for $\alpha_U(\theta,\beta) \in \left[\frac{((1-\beta)^2+2c\delta)h_\eta+\beta c\tau}{((1-\beta)^2+2c\delta)h_\eta+(1+\beta)c\tau}, 1\right]$, the right-hand side of (48) is decreasing in τ and c, and increasing in h_η and δ. Similarly, the expression $\frac{((1-\beta)^2+2c\delta)h_\eta+\beta c\tau}{((1-\beta)^2+2c\delta)h_\eta+(1+\beta)c\tau}$ is decreasing in τ and c, and increasing in h_η and δ. As a result, the function $\alpha_U(\theta,\beta)$ is decreasing in τ and c, and increasing in h_η and δ. This proves part 2 of the proposition. ■

Proof. of Corollary 1: Note that

$$\alpha_U(\theta;0) - \alpha_0 \geq \alpha_U(\theta;0) - \alpha_U(\underline{\theta};0) = \int_{\underline{\theta}}^{\theta} \frac{\partial \alpha_U(t;0)}{\partial t} dt \geq 0. \tag{49}$$

The first inequality uses that, from (46), $\alpha_U(\underline{\theta};0) = \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}$, which equals α_0 if $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. If $\frac{\tau}{2h_\eta} < \frac{1}{c}$, $\alpha_0 = \frac{8\delta h_\eta^2 + 4h_\eta\tau - c\tau^2}{8\delta h_\eta^2 + \tau(8h_\eta - c\tau)} < \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau} = \alpha_U(\underline{\theta};0)$. The last inequality is strict if $\theta > \underline{\theta}$, and it holds because $\frac{\partial \alpha_U(t;0)}{\partial t} > 0$ for any $t > \underline{\theta}$ (Part 2 of Proposition 2). ■

Proof. of Proposition 3: Note that since α_0 is independent of β, $\frac{\partial(\alpha_U(\theta;\beta)-\alpha_0)}{\partial\beta} = \frac{\partial\alpha_U(\theta;\beta)}{\partial\beta}$. Taking the derivative of (47) with respect to β at $\beta = 0$ gives:

$$\left.\frac{\partial \alpha_U(\theta;\beta)}{\partial \beta}\right|_{\beta=0} = \frac{cw\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right)}{(h_\eta(1+2c\delta)+c\tau)^2}\left(\tau(2h_\eta - c\tau) + \frac{h_\eta(h_\eta(1+2c\delta)+c\tau)(\theta-\underline{\theta})}{1+w\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right)}\right). \tag{50}$$

Note that since $-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)} \in [-\frac{1}{e}, 0)$, the value of the Lambert W function $w\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right) \in [-1, 0)$, i.e.,

$$-1 \equiv w\left(-\frac{1}{e}\right) \leq w\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right) < w(0) \equiv 0. \tag{51}$$

Therefore, $\left.\frac{\partial \alpha_U(\theta;\beta)}{\partial \beta}\right|_{\beta=0}$ has the same sign as the following expression:

$$\tau(c\tau - 2h_\eta) - \frac{h_\eta(h_\eta(1+2c\delta)+c\tau)(\theta-\underline{\theta})}{1+w\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right)}. \tag{52}$$

If $\frac{\tau}{2h_\eta} < \frac{1}{c}$, (52) is always negative because both its first term and second term are negative. The latter is true because $\theta \geq \underline{\theta}$ and $w\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right) \geq -1$. Therefore, $\left.\frac{\partial \alpha_U(\theta;\beta)}{\partial \beta}\right|_{\beta=0} < 0$. This proves part 1 of the proposition.

If $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, the first term of (52) is positive while the second term is negative. As a result, the sign of (52) can be ambiguous. Consider a limiting case of $\theta \to \underline{\theta}$:

$$\begin{aligned}
&\lim_{\theta \to \underline{\theta}} \tau(c\tau - 2h_\eta) - \frac{h_\eta(h_\eta(1+2c\delta)+c\tau)(\theta-\underline{\theta})}{1+w\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right)} \qquad (53) \\
&= \tau(c\tau - 2h_\eta) - \lim_{\theta \to \underline{\theta}} \frac{h_\eta(h_\eta(1+2c\delta)+c\tau)(\theta-\underline{\theta})}{1+w\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right)} \\
&= \tau(c\tau - 2h_\eta) - \lim_{\theta \to \underline{\theta}} \frac{h_\eta(h_\eta(1+2c\delta)+c\tau)\left[-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)} + e^{w\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right)}\right]}{e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\frac{h_\eta}{\tau}} \\
&= \tau(c\tau - 2h_\eta) \\
&\geq 0.
\end{aligned}$$

The second step uses the L' Hospital's Rule. The third step uses that

$$\lim_{\theta \to \underline{\theta}} -e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)} + e^{w\left(-e^{-\left(1+\frac{h_\eta(\theta-\underline{\theta})}{\tau}\right)}\right)} = -\frac{1}{e} + \frac{1}{e} = 0, \qquad (54)$$

where the second equality uses that $w\left(-\frac{1}{e}\right) \equiv -1$. By continuity, $\left.\frac{\partial \alpha_U(\theta;\beta)}{\partial \beta}\right|_{\beta=0} > 0$ if θ is sufficiently low. Recall that from Proposition 2, $\alpha_U(\theta;0)$ is strictly increasing in θ. Hence the condition that θ is sufficiently low is equivalent to the condition that $\alpha_U(\theta;0)$ is sufficiently low. This proves part 2 of the proposition. ■

Proof. of Lemma 2: We have proved the part of necessity in the main text of the paper. To prove sufficiency, we show that any retention schedule that satisfies conditions (1) and

(2) must be incentive compatible. Consider $\theta' > \theta$. From condition (1),

$$\int_{\theta}^{\theta'} U'(t)\,dt = \int_{\theta}^{\theta'} \left[\alpha(t) + (1-\alpha(t))\,\beta(\alpha(t))\right] dt, \tag{55}$$

and from condition (2),

$$\int_{\theta}^{\theta'} \left[\alpha(t) + (1-\alpha(t))\,\beta(\alpha(t))\right] dt \leq \int_{\theta}^{\theta'} \left[\alpha(\theta') + (1-\alpha(\theta'))\,\beta(\alpha(\theta'))\right] dt. \tag{56}$$

Therefore,

$$\int_{\theta}^{\theta'} U'(t)\,dt = U(\theta') - U(\theta) \leq \left[\alpha(\theta') + (1-\alpha(\theta'))\,\beta(\alpha(\theta'))\right](\theta' - \theta). \tag{57}$$

This proves that conditions (1) and (2) yield incentive compatibility. ■

Proof. of Proposition 4: We first prove that the solutions $\{\alpha^*(\theta), \beta^*(\theta)\}$ to program (31) also solve program (30). Note first that $\alpha^*(\theta)$ and $\beta^*(\theta^*(\alpha))$ also satisfy the IC constraint in program (30). This is because,

$$U'(\theta) = \alpha^*(\theta) + (1-\alpha^*(\theta))\,\beta^*(\theta) = \alpha^*(\theta) + (1-\alpha^*(\theta))\,\beta^*(\theta^*(\alpha^*(\theta))). \tag{58}$$

The first equality uses the IC constraint in program (31) and the second equality uses $\theta^*(\alpha^*(\theta)) = \theta$. Next, we prove that given $\alpha^*(\theta)$, the optimal choice of $\beta^*(\theta^*(\alpha))$ also maximizes the objective in program (30). Assume by contradiction, that there exists some $\{\alpha'(\theta), \beta'(\alpha)\}$ that produces higher surplus than $\{\alpha^*(\theta), \beta^*(\theta^*(\alpha))\}$ and satisfies the IC constraint in program (30). Define $\beta'(\theta) \equiv \beta'(\alpha(\theta))$. As proved previously, $\{\alpha'(\theta), \beta'(\theta)\}$

also satisfy the IC constraint in program (31). Note that $\{\alpha'(\theta), \beta'(\theta)\}$ would achieve the same amount of surplus as $\{\alpha'(\theta), \beta'(\alpha)\}$ because at every θ, $\beta'(\theta) = \beta'(\alpha(\theta))$. In addition, $\{\alpha^*(\theta), \beta^*(\theta)\}$ would achieve the same amount of surplus as $\{\alpha^*(\theta), \beta^*(\theta^*(\alpha))\}$ because at every θ, $\beta^*(\theta^*(\alpha)) = \beta^*(\theta)$, where the equality uses $\theta^*(\alpha^*(\theta)) = \theta$. Note that this implies a contradiction because at $\{\alpha'(\theta), \beta'(\theta)\}$, the surplus is lower than that at $\{\alpha^*(\theta), \beta^*(\theta)\}$, whereas at $\{\alpha'(\theta), \beta'(\alpha)\}$, the surplus is higher than that at $\{\alpha^*(\theta), \beta^*(\theta^*(\alpha))\}$.

Second, we solve for $\alpha^*(\theta)$ and $\beta^*(\theta)$. To economize on notation, we often omit the superscript "*" in the remaining proof of Proposition 4 whenever no confusion arises. At $t = 0$, the bank chooses simultaneously both the retention fraction α and the disclosure precision β. Differentiating the Hamiltonian with respect to $\alpha(\theta)$ and $\beta(\theta)$, respectively, yields:

$$\begin{aligned} & \left[\frac{1-\alpha(\theta)}{c}(1-\beta(\theta))^2 + 2\delta(1-\alpha(\theta)) - \frac{\tau[\alpha(\theta) - (1-\alpha(\theta))\beta(\theta)]}{h_\eta}\right] f(\theta) && (59) \\ = \;& L(\theta)(1-\beta(\theta)), \\ & (1-\alpha(\theta))^2 \left[\frac{1-\beta(\theta)}{c} - \frac{\tau}{2h_\eta}\right] f(\theta) && (60) \\ = \;& L(\theta)(1-\alpha(\theta)). \end{aligned}$$

Dividing (59) by (60) gives:

$$\beta(\theta) = \frac{3\alpha(\theta) - 1}{1-\alpha(\theta)} - 4\delta\frac{h_\eta}{\tau}, \tag{61}$$

if $\alpha(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$. If $\alpha(\theta) \le \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$, $\beta(\theta) = 0$ and if $\alpha(\theta) \ge \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, $\beta(\theta) = 1$. As we will verify later, the bank in equilibrium always sets $\alpha(\theta) < \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$ so the last case never prevails in equilibrium. This proves part 1 of the proposition.

Next we derive the equilibrium loan retention schedule $\alpha(\theta)$. Consider the first case that $\frac{\tau}{2h_\eta} < \frac{1}{c}$. At $\theta = \underline{\theta}$, the bank does not distort its choice, i.e., $\beta(\underline{\theta}) = \beta_0 = 1 - \frac{c\tau}{2h_\eta}$ and $\alpha(\underline{\theta}) = \frac{8\delta h_\eta^2 + 4h_\eta\tau - c\tau^2}{8\delta h_\eta^2 + \tau(8h_\eta - c\tau)} \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$. Suppose that for $\theta > \underline{\theta}$, $\alpha(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$ and we will verify this conjecture after solving the equilibrium. Substituting the expression (23) of $U(\theta)$ and $\beta(\theta) = \frac{3\alpha(\theta)-1}{1-\alpha(\theta)} - 4\delta\frac{h_\eta}{\tau}$ into the IC constraint (32) gives

$$\alpha'(\theta) = H(\alpha(\theta)), \tag{62}$$

where

$$H(x) = \frac{c\left(1 + 2\delta\frac{h_\eta}{\tau} - 2x\left(1+\delta\frac{h_\eta}{\tau}\right)\right)}{c(1-x)\left(\frac{\tau}{h_\eta}+\delta\right) - 4\left(1+\delta\frac{h_\eta}{\tau}\right)\left(1+2\delta\frac{h_\eta}{\tau} - 2x\left(1+\delta\frac{h_\eta}{\tau}\right)\right)}. \tag{63}$$

Note that the numerator of $H(x)$ is positive if and only if $x < \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$. The sign of the denominator is as follows. If $\frac{\tau}{h_\eta} < \frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c} - \delta$, the denominator of $H(x)$ is positive if and only if $x > \frac{4\left(1+\delta\frac{h_\eta}{\tau}\right)\left(1+2\delta\frac{h_\eta}{\tau}\right) - c\left(\delta+\frac{\tau}{h_\eta}\right)}{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2 - c\left(\delta+\frac{\tau}{h_\eta}\right)}$. If $\frac{\tau}{h_\eta} \geq \frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c} - \delta$, the denominator of $H(x)$ is positive for any $x \in [0,1]$.

We now prove that $\alpha(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$. First, consider the case that $\frac{\tau}{2h_\eta} < \frac{1}{c}$. At the initial point of $\alpha(\underline{\theta}) = \frac{8\delta h_\eta^2 + 4h_\eta\tau - c\tau^2}{8\delta h_\eta^2 + \tau(8h_\eta - c\tau)} \in \left(\frac{1+2c\delta}{3+2c\delta}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$, $\alpha'(\underline{\theta}) = H(\alpha(\underline{\theta})) > 0$. The schedule $\alpha(\theta)$ thus increases. As long as $\alpha(\theta)$ is below $\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$ and above $\frac{8\delta h_\eta^2 + 4h_\eta\tau - c\tau^2}{8\delta h_\eta^2 + \tau(8h_\eta - c\tau)}$, $H(\alpha(\theta)) > 0$ and $\alpha(\theta)$ keeps rising. However, $\alpha(\theta)$ can never go above $\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, because if $\alpha(\theta) = \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, $\alpha'(\theta) = H\left(\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right) = 0$ and $\alpha(\theta)$ will remain at $\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$. Therefore, $\alpha(\theta) < \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$. In addition, $\alpha(\theta) \geq \alpha(\underline{\theta}) = \frac{8\delta h_\eta^2 + 4h_\eta\tau - c\tau^2}{8\delta h_\eta^2 + \tau(8h_\eta - c\tau)} > \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$. This proves that $\alpha(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$.

Second, consider the case that $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. At $\theta = \underline{\theta}$, the bank does not distort its choice, i.e.,

$\beta\left(\underline{\theta}\right)=\beta_0=0$ and $\alpha(\underline{\theta})=\frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta+c\tau}\in\left(0,\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}\right)$. By continuity, there exists a cutoff θ_c such that for $\theta<\theta_c$, $\alpha\left(\theta\right)<\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$ and $\beta\left(\theta\right)=0$. Under $\theta<\theta_c$, plugging $\beta\left(\theta\right)=0$ into the maximization problem gives

$$\alpha'\left(\theta\right)=G\left(\alpha\left(\theta\right)\right), \tag{64}$$

where

$$G\left(x\right)\equiv\frac{c\left(1-x\right)}{x\left(1+\frac{c\tau}{h_\eta}\right)-1-2c\delta(1-x)}. \tag{65}$$

Note that upon $\beta=0$, $\alpha\left(\theta\right)\equiv\alpha_U\left(\theta;0\right)$. In addition, $G\left(x\right)>0$ if and only if $x>\alpha(\underline{\theta})=\frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta+c\tau}$. Now consider the dynamics regarding $\alpha\left(\theta\right)$ for $\theta<\theta_c$. At the initial point of $\alpha(\underline{\theta})=\frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta+c\tau}$, $\alpha'\left(\underline{\theta}\right)=G\left(\alpha(\underline{\theta})\right)>0$. The schedule $\alpha\left(\theta\right)$ thus increases. As long as $\alpha\left(\theta\right)<1$, $\alpha'\left(\theta\right)=G\left(\alpha\left(\theta\right)\right)>0$ and the schedule $\alpha\left(\theta\right)$ keeps increasing towards 1. By the intermediate value theorem, there exists a unique cutoff θ_c such that $\alpha\left(\theta_c\right)=\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$. This proves that for $\theta\leq\theta_c$, $\alpha\left(\theta\right)\leq\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$.

Next, consider the case that $\theta>\theta_c$. Suppose that for $\theta>\theta_c$, $\alpha\left(\theta\right)\in\left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}},\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$ and we will verify this conjecture after solving the equilibrium. Under this conjecture, $\beta(\theta)=\frac{3\alpha(\theta)-1}{1-\alpha(\theta)}-4\delta\frac{h_\eta}{\tau}$, and thus $\alpha'\left(\theta\right)=H\left(\alpha\left(\theta\right)\right)$. Note that the numerator of $H\left(x\right)$ is positive if and only if $x<\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$. However, under $\frac{\tau}{2h_\eta}\geq\frac{1}{c}$, the sign of $H\left(x\right)$'s denominator may be different from that under $\frac{\tau}{2h_\eta}<\frac{1}{c}$. More specifically, if $\frac{\tau}{h_\eta}<\frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c}-\delta$, $H\left(x\right)$ is positive if and only if $x\in\left(\frac{4\left(1+\delta\frac{h_\eta}{\tau}\right)\left(1+2\delta\frac{h_\eta}{\tau}\right)-c\left(\delta+\frac{\tau}{h_\eta}\right)}{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2-c\left(\delta+\frac{\tau}{h_\eta}\right)},\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$, whereas if $\frac{\tau}{h_\eta}\geq\frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c}-\delta$, $H\left(x\right)$ is positive if and only if $x<\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$. We thus discuss the cases of $\frac{\tau}{h_\eta}<\frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c}-\delta$ and $\frac{\tau}{h_\eta}\geq\frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c}-\delta$ separately.

Consider first the dynamics regarding $\alpha\left(\theta\right)$ if $\frac{\tau}{h_\eta}<\frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c}-\delta$. At the initial point of $\alpha(\theta_c) = \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$, since $\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}} \geq \frac{4\left(1+\delta\frac{h_\eta}{\tau}\right)\left(1+2\delta\frac{h_\eta}{\tau}\right)-c\left(\delta+\frac{\tau}{h_\eta}\right)}{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2-c\left(\delta+\frac{\tau}{h_\eta}\right)}$ under $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$, $\alpha'(\theta_c) = H(\alpha(\theta_c)) = H\left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}\right) > 0$. The schedule $\alpha(\theta)$ thus increases. As long as $\alpha(\theta)$ is below $\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, $H(\alpha(\theta)) > 0$ and $\alpha(\theta)$ keeps rising. However, $\alpha(\theta)$ can never go above $\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, because if $\alpha(\theta) = \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, $\alpha'(\theta) = H\left(\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right) = 0$ and $\alpha(\theta)$ will remain at $\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$. Therefore, $\alpha(\theta) < \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$. In addition, $\alpha(\theta) \geq \alpha(\theta_c') = \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$. This proves that $\alpha(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$ for $\theta > \theta_c$ if $\frac{\tau}{h_\eta} < \frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c} - \delta$ and $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$.

If $\frac{\tau}{h_\eta} \geq \frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c} - \delta$ and at the initial point of $\alpha(\theta_c') = \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$, since $\alpha(\theta_c) = \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}} < \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, $\alpha'(\theta_c) = H(\alpha(\theta_c')) = H\left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}\right) > 0$. The schedule $\alpha(\theta)$ thus increases. As long as $\alpha(\theta)$ is below $\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, $H(\alpha(\theta)) > 0$ and $\alpha(\theta)$ keeps rising. However, $\alpha(\theta)$ can never go above $\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, because if $\alpha(\theta) = \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$, $\alpha'(\theta) = H\left(\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right) = 0$ and $\alpha(\theta)$ will remain at $\frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$. Therefore, $\alpha(\theta) < \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}$. In addition, $\alpha(\theta) \geq \alpha(\theta_c) = \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$. This proves that $\alpha(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$ for $\theta > \theta_c$ if $\frac{\tau}{h_\eta} \geq \frac{8\left(1+\delta\frac{h_\eta}{\tau}\right)^2}{c} - \delta$ and $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$.

Finally, we verify the monotonicity condition, $\alpha(\theta)+(1-\alpha(\theta))\,\beta_C(\alpha(\theta))$ is non-decreasing in θ. If $\frac{\tau}{2h_\eta} < \frac{1}{c}$, since $\alpha(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$, then $\beta_C(\alpha(\theta)) = \frac{3\alpha(\theta)-1}{1-\alpha(\theta)} - 4\delta\frac{h_\eta}{\tau}$. This gives

$$\alpha(\theta) + (1-\alpha(\theta))\,\beta_C(\alpha(\theta)) = \alpha(\theta)\left(4+4\delta\frac{h_\eta}{\tau}\right) - 1 - 4\delta\frac{h_\eta}{\tau}, \tag{66}$$

which is non-decreasing in θ because $\alpha(\theta)$ is non-decreasing in θ. If $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$ and $\theta \leq \theta_c$, $\alpha(\theta) = \alpha_U(\theta;0) \in (0, \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}]$ and $\beta_C(\alpha(\theta)) \equiv 0$. This gives

$$\alpha(\theta) + (1-\alpha(\theta))\,\beta_C(\alpha(\theta)) = \alpha(\theta), \tag{67}$$

which is non-decreasing in θ because $\alpha(\theta)$ is non-decreasing in θ. If $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$ and $\theta > \theta_c$,

$\alpha(\theta) \in \left(\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right)$ and $\beta_C(\alpha(\theta)) = \frac{3\alpha(\theta)-1}{1-\alpha(\theta)} - 4\delta\frac{h_\eta}{\tau}$. This gives

$$\alpha(\theta) + (1-\alpha(\theta))\,\beta_C(\alpha(\theta)) = \alpha(\theta)\left(4 + 4\delta\frac{h_\eta}{\tau}\right) - 1 - 4\delta\frac{h_\eta}{\tau}, \tag{68}$$

which is non-decreasing in θ because $\alpha(\theta)$ is non-decreasing in θ. ■

Proof. of Corollary 2: We first prove the comparative statics of $\alpha^*(\theta)$, as defined in $\frac{\partial\alpha^*(\theta)}{\partial\theta} = H(\alpha^*(\theta))$. One can verify that if $\frac{\tau}{2h_\eta} < \frac{1}{c}$, the function $H(x)$ increases in h_η and δ, and decreases in c and τ, for $x \in \left[\frac{8\delta h_\eta^2+4h_\eta\tau-c\tau^2}{8\delta h_\eta^2+\tau(8h_\eta-c\tau)}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2+2\delta\frac{h_\eta}{\tau}}\right]$; otherwise, if $\frac{\tau}{2h_\eta} > \frac{1}{c}$, the function $H(x)$ increases in h_η and δ, and decreases in c and τ, for $x \in \left[\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2+2\delta\frac{h_\eta}{\tau}}\right]$. In addition,, if $\frac{\tau}{2h_\eta} < \frac{1}{c}$, the expression $\frac{8\delta h_\eta^2+4h_\eta\tau-c\tau^2}{8\delta h_\eta^2+\tau(8h_\eta-c\tau)}$ is decreasing in τ and c, and increasing in h_η and δ. Furthermore, both $\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$ and $\frac{1+2\delta\frac{h_\eta}{\tau}}{2+2\delta\frac{h_\eta}{\tau}}$ are decreasing in τ, and increasing in h_η and δ. These conditions jointly give that $\alpha^*(\theta)$ is decreasing in τ and c, and increasing in h_η and δ.

Next, we derive the comparative statics regarding $\beta^*(\theta)$. Substituting the optimal contingent rule $\beta^*(\theta) = \frac{3\alpha^*(\theta)-1}{1-\alpha^*(\theta)} - 4\delta\frac{h_\eta}{\tau}$, or, equivalently, $\alpha^*(\theta) = \frac{1+\beta^*(\theta)+4\delta\frac{h_\eta}{\tau}}{3+\beta^*(\theta)+4\delta\frac{h_\eta}{\tau}}$, into $\frac{\partial\alpha^*(\theta)}{\partial\theta} = H(\alpha^*(\theta))$ gives that:

$$\frac{\partial\beta^*(\theta)}{\partial\theta} = \frac{ch_\eta(1-\beta^*(\theta))}{(\delta h_\eta+\tau)(-2h_\eta+c\tau+2h_\eta\beta^*(\theta))} \times \frac{(4\delta h_\eta+3\tau+\tau\beta^*(\theta))^2}{4\tau}. \tag{69}$$

Define

$$L(x) \equiv \frac{ch_\eta(1-x)}{(\delta h_\eta+\tau)(-2h_\eta+c\tau+2h_\eta x)} \times \frac{(4\delta h_\eta+3\tau+\tau x)^2}{4\tau}. \tag{70}$$

One can verify that if $2 > \frac{c\tau}{h_\eta}$, the function $L(x)$ increases in h_η and δ, and decreases in c and τ, for $x \in \left[\frac{3\frac{8\delta h_\eta^2+4h_\eta\tau-c\tau^2}{8\delta h_\eta^2+\tau(8h_\eta-c\tau)}-1}{1-\frac{8\delta h_\eta^2+4h_\eta\tau-c\tau^2}{8\delta h_\eta^2+\tau(8h_\eta-c\tau)}} - 4\delta\frac{h_\eta}{\tau}, 1\right]$; otherwise, if $2 < \frac{c\tau}{h_\eta}$, the function $L(x)$ increases in h_η

and δ, and decreases in c and τ, for $x \in [0,1]$. Similarly, the expression $\frac{3\frac{8\delta h_\eta^2+4h_\eta\tau-c\tau^2}{8\delta h_\eta^2+\tau(8h_\eta-c\tau)}-1}{1-\frac{8\delta h_\eta^2+4h_\eta\tau-c\tau^2}{8\delta h_\eta^2+\tau(8h_\eta-c\tau)}}-4\delta\frac{h_\eta}{\tau}$ is decreasing in τ and c, and increasing in h_η and δ. These conditions jointly give that $\beta^*(\theta)$ is decreasing in τ and c, and increasing in h_η and δ.

Finally, we derive the comparative statics regarding θ_c when $\frac{\tau}{2h_\eta} > \frac{1}{c}$. From the proof of Proposition 4, when $\frac{\tau}{2h_\eta} > \frac{1}{c}$, θ_c solves $\alpha_U(\theta_c; 0) = \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$. Substituting the expression for α_U in (47) gives that:

$$\theta_c = -\frac{1}{h_\eta}\left((\tau - \underline{\theta}h_\eta + \tau log\left[2e^{-\frac{2(h_\eta+2c\delta h_\eta+c\tau)}{c(4\delta h_\eta+3\tau)}}\frac{h_\eta+2c\delta h_\eta+c\tau}{c(4\delta h_\eta+3\tau)}\right]\right). \tag{71}$$

Differentiating θ_c with respect to τ and evaluating at $\tau = \frac{2h_\eta}{c}$ gives:

$$\frac{\partial\theta_c}{\partial\tau}|_{\tau=\frac{2h_\eta}{c}} = 0. \tag{72}$$

Furthermore, we have

$$\frac{\partial^2\theta_c}{\partial\tau^2} = \frac{(3+2c\delta)h_\eta(-16\delta(1+2c\delta)h_\eta^2+4c\delta(1+6c\delta)h_\eta\tau+c(9+14c\delta)\tau^2)}{c(4\delta h_\eta+3\tau)^3(h_\eta+2c\delta h_\eta+c\tau)^2} > 0. \tag{73}$$

Hence, for any $\tau > \frac{2h_\eta}{c}$, $\frac{\partial\theta_c}{\partial\tau} > \frac{\partial\theta_c}{\partial\tau}|_{\tau=\frac{2h_\eta}{c}} = 0$. Differentiating θ_c with respect to c gives:

$$\frac{\partial\theta_c}{\partial c} = \frac{\tau(-2h_\eta+c\tau)}{c^2(4\delta h_\eta+3\tau)(h_\eta+2c\delta h_\eta+c\tau)} > 0, \tag{74}$$

given that $\frac{\tau}{2h_\eta} > \frac{1}{c}$. Differentiating θ_c with respect to δ gives:

$$\frac{\partial\theta_c}{\partial\delta} = -\frac{2\tau(-2h_\eta + c\tau)^2}{c(4\delta h_\eta + 3\tau)^2(h_\eta + 2c\delta h_\eta + c\tau)} < 0. \tag{75}$$

Differentiating θ_c with respect to h_η gives:

$$\frac{\partial\theta_c}{\partial h_\eta} = \frac{\tau}{ch_\eta^2(4\delta h_\eta + 3\tau)^2(h_\eta + 2c\delta h_\eta + c\tau)}\Big(16c\delta^2(1 + 2c\delta)h_\eta^3 + 2(3 + 2c\delta(7 + 16c\delta))h_\eta^2\tau$$
$$+ 2c(3 + 20c\delta)h_\eta\tau^2 + 9c^2\tau^3 - 2(4\delta h_\eta + 3\tau)(h_\eta + 2c\delta h_\eta + c\tau)^2$$
$$+ c(4\delta h_\eta + 3\tau)^2(h_\eta + 2c\delta h_\eta + c\tau)(\log[2(h_\eta + 2c\delta h_\eta + c\tau)] - \log[c(4\delta h_\eta + 3\tau)])\Big). \tag{76}$$

$\frac{\partial\theta_c}{\partial h_\eta} < 0$ is equivalent to

$$16c\delta^2(1+2c\delta)h_\eta^3+2(3+2c\delta(7+16c\delta))h_\eta^2\tau+2c(3+20c\delta)h_\eta\tau^2+9c^2\tau^3-2(4\delta h_\eta+3\tau)(h_\eta+2c\delta h_\eta+c\tau)^2$$
$$< c(4\delta h_\eta + 3\tau)^2(h_\eta + 2c\delta h_\eta + c\tau)\log\left[\frac{c(4\delta h_\eta + 3\tau)}{2(h_\eta + 2c\delta h_\eta + c\tau)}\right], \tag{77}$$

which is equivalent to

$$\frac{(-2h_\eta + c\tau)(4\delta(1 + 2c\delta)h_\eta^2 + 8c\delta h_\eta\tau + 3c\tau^2)}{c(4\delta h_\eta + 3\tau)^2(h_\eta + 2c\delta h_\eta + c\tau)} < \log\left[\frac{c(4\delta h_\eta + 3\tau)}{2(h_\eta + 2c\delta h_\eta + c\tau)}\right]. \tag{78}$$

We can verify that this last inequality is always satisfied when $\frac{\tau}{2h_\eta} > \frac{1}{c}$. Thus, $\frac{\partial\theta_c}{\partial h_\eta} < 0$. ■

Proof. of Corollary 3: To economize on notation, we often omit the superscript "*" in the proof of Corollary 3 whenever no confusion arises. Consider first the case in which $\frac{\tau}{2h_\eta} < \frac{1}{c}$.

Note that under no measurement, the type $\underline{\theta}$ bank chooses a non-distorted retention fraction

$$\alpha_U(\underline{\theta}; 0) = \alpha_0 = \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}. \tag{79}$$

Therefore, under no measurement, $\alpha_U(\theta) \geq \alpha_U(\underline{\theta}; 0) = \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}$. Obviously, for the set of θ in which $\alpha(\theta) < \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}$, $\alpha(\theta) < \alpha_U(\theta)$. This set is non-empty because at $\theta = \underline{\theta}$, $\alpha(\underline{\theta}) = \frac{8\delta h_\eta^2 + 4h_\eta\tau - c\tau^2}{8\delta h_\eta^2 + \tau(8h_\eta - c\tau)} < \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}$ under $\frac{\tau}{2h_\eta} < \frac{1}{c}$. Consider now the set of θ in which $\alpha(\theta) \geq \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}$. Since $\alpha(\theta)$ is non-decreasing in θ, by continuity, there exists a cutoff θ_Λ such that $\alpha(\theta) \geq \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}$ if and only if $\theta \geq \theta_\Lambda$, where $\alpha(\theta_\Lambda) = \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}$. Note that $\theta_\Lambda > \underline{\theta}$ because $\alpha(\theta_\Lambda) = \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau} > \alpha(\underline{\theta}) = \frac{8\delta h_\eta^2 + 4h_\eta\tau - c\tau^2}{8\delta h_\eta^2 + \tau(8h_\eta - c\tau)}$; in addition, $\alpha_U(\theta_\Lambda; 0) > \alpha_U(\underline{\theta}; 0) = \frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau} = \alpha(\theta_\Lambda)$. Furthermore, one can verify that for all $\alpha \in \left[\frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}, 1\right]$, $H(\alpha) < G(\alpha)$. Recall also that $\alpha'(\theta) = H(\alpha(\theta))$ and $\alpha_U'(\theta; 0) = G(\alpha_U(\theta; 0))$. Thus, for any $\theta \geq \theta_\Lambda$, the slope of $\alpha_U(\theta; 0)$ is higher than the slope of $\alpha(\theta)$. This implies that for any $\theta \geq \theta_\Lambda$, $\alpha_U(\theta; 0) > \alpha(\theta)$ because at the initial point of $\theta = \theta_\Lambda$, $\alpha_U(\theta_\Lambda; 0) > \alpha(\theta_\Lambda)$ and $\alpha_U(\theta; 0)$ increases at a faster speed than $\alpha(\theta)$. This proves part 1 of the corollary.

Next, consider that $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. From Proposition 4, the initial condition at $\theta = \theta_c$ is $\alpha(\theta_c) = \alpha_U(\theta_c; 0) = \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$. Furthermore, one can verify that for all $\alpha \in \left[\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1+2\delta\frac{h_\eta}{\tau}}{2\left(1+\delta\frac{h_\eta}{\tau}\right)}\right]$, $H(\alpha) < G(\alpha)$. This implies that for all $\theta > \theta_c$, $\alpha(\theta) < \alpha_U(\theta; 0)$ because at the initial point of $\theta = \theta_c$, $\alpha_U(\theta_c; 0) = \alpha(\theta_c)$ and $\alpha_U(\theta; 0)$ increases at a faster speed than $\alpha(\theta)$. In addition, from Proposition 4, for all $\theta \leq \theta_c$, $\alpha(\theta) = \alpha_U(\theta; 0)$. This proves part 2 of the corollary. ■

Proof. of Corollary 4: Consider first the case in which $\frac{\tau}{2h_\eta} < \frac{1}{c}$. Note that under no measurement, the type $\underline{\theta}$ bank chooses a monitoring effort that equals:

$$m_U(\underline{\theta}) = \frac{\alpha_0}{c} = \frac{1}{c}\frac{(1+2c\delta)h_\eta}{(1+2c\delta)h_\eta + c\tau}, \tag{80}$$

whereas under optimal measurement, the type $\underline{\theta}$ bank chooses a monitoring effort that equals:

$$m(\underline{\theta}) = \frac{\alpha_0 + (1-\alpha_0)\beta_0}{c} = \frac{1}{c}\frac{8\delta h_\eta^2 + 8h_\eta\tau - 3c\tau^2}{8\delta h_\eta^2 + 8h_\eta\tau - c\tau^2}. \tag{81}$$

Note that under $\frac{\tau}{2h_\eta} < \frac{1}{c}$, $m(\underline{\theta}) > m_U(\underline{\theta})$. For $\theta > \underline{\theta}$, $m_U(\theta) = \frac{\alpha_U(\theta;0)}{c}$, where $\alpha_U(\theta;0)$ solves $\alpha_U'(\theta;0) = G(\alpha_U(\theta;0))$. Replacing $\alpha_U(\theta;0) = cm_U(\theta)$ gives that

$$m_U'(\theta) = \frac{1}{c}G(cm_U(\theta)). \tag{82}$$

Similarly, for $\theta > \underline{\theta}$,

$$m(\theta) = \frac{\alpha(\theta) + (1-\alpha(\theta))\beta(\theta)}{c} = \frac{4\left(1+\delta\frac{h_\eta}{\tau}\right)\alpha(\theta) - 1 - 4\delta\frac{h_\eta}{\tau}}{c}. \tag{83}$$

The last equality uses that $\beta(\theta) = \frac{3\alpha(\theta)-1}{1-\alpha(\theta)} - 4\delta\frac{h_\eta}{\tau}$. Note that $m(\theta)$ is non-decreasing in θ since $\alpha(\theta)$ is non-decreasing. Therefore, $m(\theta) > m(\underline{\theta}) = \frac{1}{c}\frac{8\delta h_\eta^2+8h_\eta\tau-3c\tau^2}{8\delta h_\eta^2+8h_\eta\tau-c\tau^2}$. Replacing $\alpha(\theta) = \frac{cm(\theta)+\left(1+4\delta\frac{h_\eta}{\tau}\right)}{4\left(1+\delta\frac{h_\eta}{\tau}\right)}$ gives that

$$m'(\theta) = \frac{4\left(1+\delta\frac{h_\eta}{\tau}\right)}{c}H\left(\frac{cm(\theta)+\left(1+4\delta\frac{h_\eta}{\tau}\right)}{4\left(1+\delta\frac{h_\eta}{\tau}\right)}\right). \tag{84}$$

Furthermore, one can verify that for all $m \in \left[\frac{1}{c}\frac{8\delta h_\eta^2+8h_\eta\tau-3c\tau^2}{8\delta h_\eta^2+8h_\eta\tau-c\tau^2}, \frac{1}{c}\right]$, $\frac{4\left(1+\delta\frac{h_\eta}{\tau}\right)}{c}H\left(\frac{cm(\theta)+\left(1+4\delta\frac{h_\eta}{\tau}\right)}{4\left(1+\delta\frac{h_\eta}{\tau}\right)}\right) > \frac{1}{c}G\left(cm_U\left(\theta\right)\right)$. This implies that for all θ, $m\left(\theta\right) > m_U\left(\theta\right)$ because at the initial point of $\theta = \underline{\theta}$, $m\left(\underline{\theta}\right) > m_U\left(\underline{\theta}\right)$ and $m(\theta)$ increases at a faster speed than $m_U\left(\theta\right)$. This proves part 1 of the corollary.

Next, consider that $\frac{\tau}{2h_\eta} \geq \frac{1}{c}$. From Proposition 4, the initial condition at $\theta = \theta_c$ is $\alpha(\theta_c) = \alpha_U(\theta_c; 0) = \frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$ and $\beta\left(\theta_c\right) = 0$. Thus at $\theta = \theta_c$, $m_U\left(\theta_c\right) = m\left(\theta_c\right) = \frac{1}{c}\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}$. Furthermore, one can verify that that for all $m \in \left[\frac{1}{c}\frac{1+4\delta\frac{h_\eta}{\tau}}{3+4\delta\frac{h_\eta}{\tau}}, \frac{1}{c}\right]$, $\frac{4\left(1+\delta\frac{h_\eta}{\tau}\right)}{c}H\left(\frac{cm(\theta)+\left(1+4\delta\frac{h_\eta}{\tau}\right)}{4\left(1+\delta\frac{h_\eta}{\tau}\right)}\right) > \frac{1}{c}G\left(cm_U\left(\theta\right)\right)$. This implies that for all $\theta > \theta_c$, $m\left(\theta\right) > m_U\left(\theta\right)$ because at the initial point of $\theta = \theta_c$, $m\left(\theta_c\right) = m_U\left(\theta_c\right)$ and $m(\theta)$ increases at a faster speed than $m_U\left(\theta\right)$. In addition, from Proposition 4, for all $\theta \leq \theta_c$, $\alpha(\theta) = \alpha_U(\theta; 0)$ and $\beta\left(\theta\right) = 0$. Therefore, $m_U\left(\theta\right) = m\left(\theta\right) = \frac{\alpha_U(\theta;0)}{c}$. This proves part 2 of the corollary. ■