Abstract

Within the spokes model of Chen and Riordan (2007) that allows for non-localized competition among arbitrary numbers of commercial and non-commercial media outlets, this paper studies the quality and accuracy of media content under different ownership structures and market environments. A main result shows that too few commercial media outlets, or better, too few separate owners of commercial outlets can lead to substantial bias. Adding more owners but also adding non-commercial media outlets brings down the bias; to a lesser extent, so does adding outlets given a fixed number of owners. The paper provides fresh arguments against media consolidation.

Keywords: Media consolidation; commercial media; efficient media ownership structure; self-censorship; media bias. JEL Classification: L13; L82.
“Papa, what is the moon supposed to advertise?”

Carl Sandburg, THE PEOPLE, YES, 1936

(cited from Barnouw, 1978, p. 3)

1 Introduction -- incomplete

Motivated by the recent media policy debate in the United States and ongoing attempts by the Federal Communications Commission (FCC) to loosen ownership rules there (see e.g., McChesney, 2004, for a description of events around the 2003 attempt), we develop a model of media competition that allows a detailed study of the quality and bias of media content for a number of different ownership structures. Our analysis builds on the spokes model of Chen and Riordan (2007), which is a Hotelling type model of spatial competition that allows for arbitrary numbers of media firms to compete against each other in a non-localized fashion. Within this framework, we consider two types of media outlets, namely, (i) commercial outlets, which maximize profits and are financed mainly through advertising, and (ii) non-commercial outlets, which are funded by either viewers’ fees or an exogenously given budget.

Our main results show that too few separately owned commercial media firms can lead to substantial bias of their media content. How many firms are “too few” depends on certain demand, cost, and preference parameters that can in principle be estimated empirically. Increasing the number of separately owned media firms and increasing the number of non-commercial outlets helps towards reducing the biases; and increasing the number of commercial outlets in the market, while keeping the number of owners fixed, can also help, but clearly to a lesser extent.

More specifically, the basic model is structured as follows. There are $n$ ($\geq 2$) commercial media outlets and a mass one of consumers. The media outlets are located at the endpoints (one for each outlet) of a spokes network that has $N \geq n$ potential locations. Any two endpoints have distance 1 from each other (and each endpoint therefore has distance $\frac{1}{2}$ to the center of the
network). Commercial media outlets are assumed to maximize profits which are derived from advertising and payments from the audience minus the costs of producing the programming. Later on (in Section 3.4), we consider the important case where commercial media firms can own multiple ($\kappa \geq 1$) outlets, in which case the $n\kappa$ total outlets are located (as before) at $n\kappa$ different endpoints of a spokes network with $N \geq \kappa n$ endpoints; and finally (in Sections 4 and 5) we also allow for the possibility of $m \geq 0$ non-commercial media outlets to be in the market, again each one located at a different endpoint of a spokes network with $N \geq n + m$ endpoints.

The consumers are uniformly distributed along the $N$ spokes of the network. Transportation (or switching) costs are equal for all consumers in the sense that there is a constant cost $t > 0$ to “travel” a unit distance (or to “switch” to another outlet). As in Chen and Riordan (2007), consumers have a preference for only two of the $N$ potential outlets. One interpretation of this is that at a given time of the day a consumer has a preference for only two outlets and any other media outlet does not come into question, possibly because there is a non-media outside option that is preferred. When $N > n + m$ then this means that there are some consumers who are interested in consuming exactly two, one or zero of the $n + m$ actual outlets.

Besides the assumptions implicit in the framework, the analysis relies on three key assumptions: (A1) Total advertising revenues are a constant fraction ($\eta \geq 0$) of final sales by the advertisers ($C$); (A2) advertisers advertise with all commercial media outlets in proportion to their share of the audience ($s_i \geq 0$); (A1 and A2 combined imply that media outlet $i$’s advertising revenues can be written as $s_i \eta C$). (A3) final sales can be written as $C(x) = \varphi(x)C_0$, where $\varphi$ is a decreasing function of the sensitive information variable $x = (x_1, \ldots, n)$ and $C_0$ is exogenously given base consumption of the branded products.

The justifications for the three assumptions are mainly empirical. For (A1), Schmalensee (1972) derives a constant fraction rule as the optimal amount of advertising under several different oligopoly settings and provides some empirical support; for more empirical evidence see also Baghestani
(1991), Jung and Seldon (1995), Elliott (2001); as well as Esteve and Requena (2006) for some qualifications; for (A2), see, for example, Advertising Age 2007; for (A3) the information variable $x$ is essentially defined through this assumption (see also the discussion in Section 2). Consumers might be put off for a variety of reasons that can include direct health concerns or for ethical, political or ideological reasons. To the extent that branded or advertised products are not “ethical,” such information will be sensitive. On the other hand, once more “ethical” substitutes are advertised, it ceases being sensitive, and information that once depressed consumption may well boost consumption of the (newly) advertised goods. Clearly what constitutes sensitive information may vary over time.

Given these assumptions, we derive a critical number $\bar{n}$ such that if there are fewer than $\bar{n}$ firms in the market, then in equilibrium sensitive information can be completely suppressed ($x_i = 0$ for all $i$) by all firms; on the other hand, a sufficiently large number of outlets will always guarantee zero suppression or maximum accuracy ($x_i = 1$ for all $i$) (Proposition 1). The result seems to be robust to different ownership structures, like allowing media firms to own multiple outlets (Proposition 5) and to including non-commercial media outlets (Proposition 8). While audience-funded commercial media tend to be more informative, the possibility of drawing revenues from advertisers can cancel this effect (Propositions 3 and 4). Finally, when allowing for free entry, we also point out that the market may not support sufficiently many separate firms in equilibrium if fixed costs are too high. On the other hand, introducing non-commercial outlets can bring down the critical number $\bar{n}$ of commercial firms needed to avoid commercially driven suppression of sensitive information (Proposition 8). Finally, the framework also allows media outlets to choose a further quality variable ($y$) that is separate from the accuracy on the sensitive topics just mentioned.

Some remarks on our notion of media bias (measured by the variable $x$) are in order. In a strict sense, the variable $x$ can be thought of as measuring the amount of information provided on “sensitive” topics (defined implicitly through the effect on $C(x)$), so that $x = 0$ corresponds to minimum
accuracy or full suppression (self-censorship), while at the other end \( x = 1 \) corresponds to full information provision or maximum accuracy or absence of suppression (no censorship). Two well-documented examples that illustrate this are the (non)reporting of the health hazards of smoking (see e.g., Baker, 1994, and Bagdikian, 2004, for chronologies and also the discussion in Ellman and Germano, 2008) and on the case for anthropogenic climate change (see e.g., Boykoff and Boykoff, 2004, for an evaluation of recent coverage in the US quality press and contrast with Oreskes, 2004, which evaluates the scientific benchmark). Both are examples with highly nontrivial individual or global consequences.

According to our model, a main reason for suppressing coverage on, say, tobacco and climate change is the presence of large advertisers\(^1\) and the latter’s interest in minimizing content in mainstream media that might adversely affect sales of their products. As the journalism, media and communications literatures show, there are more examples of systematic underreporting in this sense (see, e.g., Bagdikian, 2004, Baker, 1994, Barnouw, 1978). At the same time, the information variable \( x \) can also be thought of as representing “critical” content in the sense of the inverse of “dumbed down,” neutralized or “watered down” content (as discussed e.g., in Hamilton, 2004, McChesney, 2000, 2004) or as the inverse of excessive consumerism-oriented content (e.g., Baker, 1994, Bagdikian, 2004, McChesney, 2000, 2004). The idea is that also here, “critical” (or less “dumbed down”) content (larger \( x \)) tends to lead to less consumption of the advertised products, which in turn, through the internalization by the commercial media, is the source of their biased coverage in this respect.

Related literature. Ellman and Germano (2008) is particularly closely related as it models the effect of advertiser influence on media content. Unlike, the present paper it explicitly models both advertisers and consumers (besides the media outlets), and so to some extent provides microfoundations for some of the assumptions of the current model. The location model as-

\(^1\)Tobacco companies such as Brown & Williamson (part of British American Tobacco) or Philip Morris (now Altria Group) and car manufacturers such as General Motors or Ford have consistently been among top advertisers in the US at different points in time.
pect of the present paper, on the other hand, allows to get a more tractable and possibly also more realistic picture of the case of multiple media outlets \((n \geq 2)\) and with different ownership structures. We believe nonetheless that the present paper underestimates the effects of (large) advertisers to influence media content as it does not explicitly model threats from advertisers to withdraw their ads from commercial media.

Also very closely related is Armstrong and Weeds (2007) who use a Hotelling model (for the case \(n = 2\)) and a Salop model (for the case \(n \geq 2\)) to evaluate the role of public broadcasting and pay TV on the quality of programming.


The paper is organized as follows. Section 2 sets out the basic model that is used throughout the paper. Sections 3 and 4 consider environments with commercial and noncommercial media outlets respectively; Section 5 allows for mixed environments and Section 6 studies the issue of entry. Section 7 briefly discusses issues of welfare and Section 8 concludes with a discussion of policy implications. Most proofs and derivations are contained in an appendix.

## 2 The basic framework

We work with the spokes model of Chen and Riordan (2007) that allows for an arbitrary number of media outlets to compete for audience in a non-localized fashion – unlike the Salop (1979) model. The model we develop shares important features with many other papers in the media literature that have worked with the Hotelling model (when \(n = 2\)) or the Salop model.
There are $N$ potential brands and $n$ actual ones. Each consumer has a preference for (at most) two potential brands so that if $N > n$ some consumers may have a preference for one or zero actual brands. (In a separate paper, we study a particular case of this, namely, where $N = n$. In that case the market is “covered” and each consumer has a preference for exactly two brands, namely, the brand corresponding to the spoke the consumer is located on and another brand chosen at random and with uniform probability from all the remaining spokes. Consumers are therefore indifferent or close to indifferent between the two brands, both of which are available on the market. Any two firms compete for such consumers that are approximately indifferent between their products, while at the same time competing with other firms for other consumers. Notice that this simultaneous competition on several fronts is what distinguishes this spokes model with Salop’s (1979) model, where a given firm competes essentially with its two neighbors.

There is an exogenous degree of horizontal product differentiation between the media outlets. Shares are determined by the equation

$$s_i = \frac{n - 1}{N(N - 1)} + \frac{1}{N(N - 1)} \sum_{j \neq i} s_{ij} + \frac{2(N - n)}{N(N - 1)},$$

where $s_{ij}$ is the share of viewers on $j$’s spoke that $i$ appropriates from $j$, that is

$$s_{ij} = \begin{cases} 
-1 & \text{if } u_i - u_j < -t \\
\frac{u_i - u_j}{t} & \text{if } |u_i - u_j| \leq t \\
1 & \text{if } u_i - u_j > t
\end{cases},$$

where $t > 0$ is a transportation cost and outlet $i$’s utility is given by

$$u_i = v + \alpha x_i + \beta y_i - p_i,$$

and define $\bar{u}_i = \frac{1}{n} \sum_i u_i$. Also, $v \gg \alpha + \beta$ is the exogenous valuation of consuming the media; $\alpha, \beta \geq 0$ are parameters; $x_i, y_i \in [0, 1]$, where $x_i$ is accuracy or amount reported on sensitive topic; $y_i$ is an endogenous measure of quality; $p_i \geq 0$ is the price charged by $i$. Assuming $|u_i - u_j| \leq t$, which
we assume unless otherwise stated, the share equation reduces to

\[ s_i = \frac{n - 1}{N(N - 1)} + \frac{1}{N(N - 1)t} \sum_{j \neq i} (u_i - u_j) + \frac{2(N - n)}{N(N - 1)} \]

\[ = \frac{A_n}{n} + \frac{n(u_i - \bar{u})}{N(N - 1)t}, \]

where, given our assumption on the exogenous values \( v >> 0 \), we can define total audience reached by the \( n \) actual outlets in a market with \( N \) potential outlets as

\[ A_n = n \left( \frac{n - 1}{N(N - 1)} + \frac{2(N - n)}{N(N - 1)} \right) = \frac{n(2N - n - 1)}{N(N - 1)}, \tag{1} \]

Aggregate demand for the advertisers’ products is given by

\[ C(x) = C_0 \cdot e^{-\psi \sum_{i=1}^{n} s_i x_i}, \tag{2} \]

where \( x = (x_1, \ldots, x_n) \) is what we refer to as the information or “accuracy” variable, and \( \psi \in [0, 1] \) is a constant representing the marginal effect of information available to the audience on final consumption.

The more information or the more accurate the information that reaches the public, which we represent with a larger value of \( x \), the less the advertised products will be consumed; moreover, \( x \) decreases consumption at a decreasing rate. The functional form is chosen mainly for analytic tractability (Germano and Meier, 2008, work with a linear version, and obtain qualitatively comparable results with respect to this).

As discussed in the introduction, while we take Eq. (2) as the defining characteristic of the information variable \( x \), we believe it can represent a variety of different things. For example, it could represent information that a product does direct harm to a consumer (as in the case of tobacco; see also the Monsanto example; but also dangerous toys, cars, fattening foods, drinks; pharmaceutical products with potentially serious side effects); it could be information that a product is made in a possibly non-desirable way (e.g., with genetically modified organisms GMO’s) or under non-desirable conditions (e.g., that are in violation of basic standards such as the the Ethical
Trading Initiative, ETI). In each case, the information may put off some consumers from buying a product and hence decrease the total amount of the “generic” advertised good demanded. It could also represent “critical” content in the sense of the inverse of “dumbed down” content (as discussed in Hamilton, 2004, Ellman and Germano, 2008) or as the inverse of excessive consumerism-oriented content (see e.g., Baker, 1994, Bagdikian, 2004, McChesney, 2004). More “critical” (or less “dumbed down”) content tends to lead to less consumption of the advertised products. Baron (2006) and Anderson and McLaren (2007) show how advertisers can benefit from suppressing information in a world with Bayes-rational audiences; see also Ellman and Germano (2008) for a related derivation within the present context.

3 Commercial media

We assume commercial media maximize profits, which consist of revenues from advertising and from fees paid by the audience minus the costs of producing the programming. Specifically, a given commercial media outlet $i$ maximizes profits, which are given by,

$$\pi_i = s_i \eta C + s_i p_i - \frac{\delta}{2} y_i^2,$$

where, since we assume a mass one of consumers, $C$ is at the same time total and per capita consumption of the advertised products, $p_i$ is at the same time the revenue from all consumers as well as the price paid by an individual consumer to access outlet $i$, and $y_i$ is the level of quality of the programming that is accessed equally by all consumers who have access to the outlet; $0 < \eta < 1$, $\delta > \frac{\eta C_0}{n}$ are fixed parameters, where the assumption on $\delta$ ensures that $y_i \in [0, 1]$. The first and second expressions represent revenues from advertising and from the audience respectively. As discussed in the introduction, we assume that advertising revenues are a fixed share of total sales of the advertised products weighted by the audience share of the outlet. (See Schmalensee, 1972, Baghestan, 1991, Seldon and Jung, 1996, Elliott, 2001, for theoretical and empirical support for this assumption; but see also Esteve and Requena, 2006, for some qualifications.) The third expression
represents the costs of producing quality of programming. This can be viewed as a fixed cost to produce a programming of quality $y_i$; for simplicity we also assume that providing information $x_i$ is costless.

### 3.1 Advertising funded media

We first consider purely advertising funded media where by assumption prices are zero ($p_i = 0$). By inspection of the marginal profits with respect to $x_i$

$$\frac{\partial\pi_i}{\partial x_i} = \left( \frac{\alpha(n-1)}{N(N-1)t} - \frac{\psi A_n^2}{n^2} \right) \eta C_0 e^{-\psi A_n x}$$

we see that profits are increasing whenever $\alpha(n-1) > \frac{\psi t (2N-n-1)^2}{N(N-1)}$ and decreasing whenever $\alpha(n-1) \leq \frac{\psi t (2N-n-1)^2}{N(N-1)}$, so that, when the first inequality holds, it is best to set the maximum level of accuracy ($x = 1$), while it is best to set the minimum level ($x = 0$) whenever the second inequality holds.

In other words, solving the equality

$$\frac{n-1}{(2N-n-1)^2} = \frac{\psi t}{\alpha N(N-1)}$$

with respect to $n$, gives the critical number of media outlets

$$\bar{n} = 2N - 1 - \frac{N-1}{2\psi t} \left( \sqrt{(\alpha N)^2 + \alpha N 8\psi t} - \alpha N \right), \quad (3)$$

at which the optimal strategy switches from minimum to maximum accuracy. It can be shown that for $n < \bar{n}$ we have $x = 0$ as the unique equilibrium solution; while for $n > \bar{n}$ we have $x = 1$. This leads to the first main result.

**Proposition 1** In a market with $N$ potential media outlets and $n < N$ actual purely advertising funded outlets there is a unique equilibrium with minimum accuracy ($x = 0$) whenever $n \leq \bar{n}$, and with maximum accuracy ($x = 1$) whenever $n > \bar{n}$, where $\bar{n}$ is given in Eq. (3) above.

Too heavily concentrated media markets or too few media outlets ($n \leq \bar{n}$) lead to substantial bias ($x = 0$). Lower transportation costs ($t$) and a low marginal effect of sensitive information on consumption ($\psi$) tend to relax
Figure 1: Plots of $\bar{n}$ as function of $\alpha$ for $t = 1$ (black), $t = 5$ (dark grey) and $t = 10$ (light grey) and $[N = 100, \psi = .1]$; the areas above the curves $\bar{n}$ are the combinations that avoid censorship (for $t = 1, 5, 10$).

the constraint on the number of outlets needed to avoid substantial bias (see Figure 1), while a lower preference parameter on sensitive issues ($\alpha$) (which might in turn be induced by low “awareness” of these issues) tightens the constraint.

Solving for the equilibrium amount of quality to provide, we obtain

$$\frac{\partial \pi_i}{\partial y_i} = \eta \left( \frac{\beta(n - 1)}{N(N - 1)t} C_0 e^{-\psi A_n x} \right) - \delta y = 0,$$

which implies

$$y = \frac{\eta \beta(n - 1)}{\delta N(N - 1)t} C_0 e^{-\psi A_n x},$$

so we can state the following.

**Proposition 2** Under the conditions of Proposition 1, the optimal quality levels are given by

$$y = \frac{\beta(n - 1)}{\delta N(N - 1)t} \eta C_0$$ if $n \leq \bar{n}$ and

$$y = \frac{\beta(n - 1)}{\delta N(N - 1)t} \eta C_0 e^{-\psi A_n x}$$ if $n > \bar{n},$ where $A_n$ is defined in Eq. (1).

In particular, in both cases, the level of quality ($y$) is increasing in the number of actual media outlets ($n$) and in the preference parameter for quality ($\beta$), and is decreasing in the transportation costs ($t$).
Figure 2: Plots of $x$ (solid) and $y$ (dashed) as functions of $n$ for $t = 1$ (grey) and $t = 5$ (black) and $[N = 100, \beta = 1, \psi = .1]$; higher transportation costs reduce accuracy and quality.

### 3.2 Audience funded media

Next consider media outlets funded exclusively by the audience who pay a price $p_i$ for accessing media outlet $i$ and assume (for now) $\eta = 0$. From the symmetric FOC’s we immediately get $x = 1$ and for the quality and price we get

$$
\frac{\partial \pi}{\partial y_i} = p \frac{\beta(n-1)}{N(N-1)t} - \delta y = 0, \quad \frac{\partial \pi}{\partial p_i} = \frac{A_n}{n} - p \frac{n-1}{N(N-1)t} = 0,
$$

with $A_n$ defined in Eq. (1), which gives

$$
p = \frac{(2N - n - 1)t}{n-1} \quad \text{and} \quad y = \frac{\beta(2N - n - 1)}{\delta N(N-1)}
$$

as the equilibrium price and quality.

**Proposition 3** In a market with $N$ potential media outlets and $n < N$ actual purely audience funded media outlets there is a unique equilibrium with maximal accuracy ($x = 1$) at positive prices for any number of media outlets.

Here the price and the quality both decrease as the number of actual media outlets in the market increases; the price decreases as the transportation cost decreases. More intense competition decreases prices and releases fewer funds to provide quality.
3.3 Audience and advertising funded media

Before considering multiple ownership, we briefly consider the general case where outlets can obtain revenues from both advertising and directly from the audience. Solving for equilibria where all media outlets choose simultaneously accuracy on the sensitive topic $x_i$, quality $y_i$, and prices charged $p_i$, we obtain from the FOC’s, after imposing symmetry,

$$p = \left[ \frac{(2N-n-1)t}{n-1} - \eta C_0 e^{-\psi A_n x} \right]^+$$

and

$$y = \frac{\beta(n-1)}{\delta N(N-1)t} \eta C_0 e^{-\psi A_n x}.$$

From here we see that whenever transportation costs are too large, $t \leq \frac{n-1}{2N-n-1} \eta C_0 e^{-\psi A_n}$, prices are automatically set to zero, in which case the equilibria reduce to the ones studied in Section 3.1.

**Proposition 4** If transportation costs are not excessively large, namely, $t \leq \frac{n-1}{2N-n-1} \eta C_0 e^{-\psi A_n}$, then the media outlets will choose not to charge their audience ($p = 0$) and so will be exclusively advertising funded. The equilibrium levels of bias and quality will coincide with the ones of Propositions 1 and 2.

We interpret this as saying that unless there are very large transportation costs (as can occur e.g., with a single live event such as a world cup soccer final) or very small contributions from advertising (e.g., $\eta$ or $C_0$ very small) media outlets refrain from charging their audiences. We take the zero price case as the more relevant one and do not fully pursue the case of positive prices.

3.4 Multiple ownership

We next turn to the important case where media firms can own more than one media outlet. Let $n$ now denote the total number of owners, and suppose each owner owns $\kappa \geq 1$ actual media outlets. The owner’s profits are then

$$\pi_i = \sum_{\kappa'_{i}=1}^{\kappa_i} \left( \frac{s_{\kappa'_{i} \kappa_{i}}}{} \eta C - \frac{\delta}{2} \frac{y^2}{\kappa'_{i}} \right),$$

and total consumption is

$$C = C_0 \cdot e^{-\psi \sum_{i=1}^{n} \sum_{\kappa'_{i}=1}^{\kappa_i} \frac{s_{\kappa'_{i} \kappa_{i}} x_{\kappa'_{i} \kappa_{i}}}}.$$
Defining total audience reached as
\[ A_{kn} = \frac{(2N - \kappa n - 1)\kappa n}{N(N-1)}, \]
we can compute firm \( i \)'s marginal profits with respect to \( x_{\kappa''} \) as
\[
\frac{\partial \pi}{\partial x_{\kappa''}} = \left( \frac{\alpha \kappa(n - 1)}{N(N-1)t} - \frac{\psi A_{kn}^2}{\kappa n^2} \right) \eta C_0 e^{-\psi A_{kn} x}.
\]
Again, these are either positive or negative depending on whether the number of separately owned firms \( n \) is smaller or greater than the critical value \( \bar{n}(\kappa) \) given by
\[
\bar{n}(\kappa) = 1 + \frac{\psi t}{\alpha} \frac{N - 1}{N} \quad \text{and} \quad \bar{\kappa}_{\text{max}} = \frac{\alpha N^2}{\alpha N + \psi(N-1)t}.
\]
Depending on the values of the parameters \( \alpha, \psi \) and \( t \), the value \( \bar{\kappa}_{\text{max}} \) can be substantially below the benchmark value of \( N \), which is the maximum number of outlets a media firm can own while avoiding the censorship problem,

\footnote{It is worth noting here that in the “covered” version of Chen and Riordan (2007), where \( \kappa n = N \) and where increasing the number of outlets a media firm is allowed to own automatically increases the total number of spokes, the function \( \bar{n}(\kappa) \) actually increases with \( \kappa \). This means that adding additional outlets actually increases the chances of having full censorship. See Germano and Meier (2008) for the details.}
Figure 3: Plot of $\bar{n}(\kappa)$ (grey), $\frac{N}{\kappa}$ (black) and $\frac{n(1)}{\kappa}$ (dashed) as a function of $\kappa$ for $[t = 5, N = 200, \alpha = \psi = .1]$ as well as $\bar{n}(1) \approx 20$ and $\bar{n}_{\text{min}} \approx 6$ (both dashed); the area above $\bar{n}(\kappa)$ (grey) and below $\frac{N}{\kappa}$ (black) are all the feasible combinations that avoid censorship; they intersect at $(\bar{\kappa}_{\text{max}}, \bar{n}_{\text{min}}) \approx (33, 6)$.

if what mattered were only the number of outlets; and similarly the number $\bar{n}_{\text{min}}$ substantially above 1. Let $\bar{n} = \bar{n}(1)$, then we can state our second main result, namely the multiple ownership version of Proposition 1.

**Proposition 5** In a market with $N$ potential media outlets, $n \geq 2$ media firms, each of which owns the same number $\kappa \geq 1$ of purely advertising funded media outlets, and where $\kappa n < N$, there is a unique equilibrium with minimum accuracy ($x = 0$) whenever $n \leq \bar{n}(\kappa)$ and a unique equilibrium with maximum accuracy ($x = 1$) whenever $n > \bar{n}(\kappa)$, where $\bar{n}(\kappa)$ is given in Eq. (4) and satisfies $\bar{n}(\kappa) > \frac{n(1)}{\kappa}$ with strict inequality whenever $\kappa > 1$.

Moreover, a minimum of $\bar{n}_{\text{min}} > 1$ separate media owners (with $\bar{\kappa}_{\text{max}}$ outlets each) are necessary to avoid minimum accuracy, where $\bar{n}_{\text{min}}$ (and $\bar{\kappa}_{\text{max}}$) are defined in Eq. (5).

Allowing media firms to own multiple media outlets can help but does not solve the censorship problem. This is best seen in Figure 3, where the grey curve ($\bar{n}(\kappa)$) plots the number of owners necessary to avoid the censorship problem.

The result also shows that the numbers $\bar{n}(\kappa)$ are to an important extent about the number of owners rather than the number of media outlets. To see
Figure 3 and compare the grey curve ($\bar{n}(\kappa)$) with the dashed curve ($\frac{N}{n}$) which represents the benchmark, where what matters is only the number of media outlets. The discrepancy becomes larger for smaller values of $\alpha$ or larger values of $t$. The intuition stems essentially from the fact that having multiple media outlets gives the media firms additional monopoly power, which results in additional censorship.\footnote{Notice that with $n$ owners each of which owns $\kappa$ media outlets, the share of consumers that are “trapped” between two firms of a single owner is $\frac{\kappa - 1}{\kappa n - 1}$, which goes from 0 to $\frac{1}{n}$ as $\kappa \to \infty$. check!}

From the FOC’s, using symmetry, we compute,
\[
\frac{\partial \pi}{\partial y_{\kappa_i}} = \frac{\beta \kappa (n-1)}{N(N-1)t} \left( \eta C_0 e^{-\psi A_{\kappa n} x} + p \right) - \delta y_{\kappa_i},
\]
\[
\frac{\partial \pi}{\partial p_{\kappa_i}} = \frac{A_{\kappa n}}{\kappa n} - \frac{\kappa (n-1)}{N(N-1)t} \eta C_0 e^{-\psi A_{\kappa n} x} - \frac{\kappa (n-1)}{N(N-1)t} \delta p,
\]
which leads to the equilibrium prices and quality levels,
\[
p = \left[ \frac{(2N - \kappa n - 1)t}{\kappa (n-1)} - \eta C_0 e^{-\psi A_{\kappa n} x} \right]^+ = 0, \quad y = \frac{\beta \kappa (n-1)}{\delta N(N-1)t} \eta C_0 e^{-\psi A_{\kappa n} x}.
\]

Again, increasing the number of owners ($n$) or of subsidiary outlets per owner ($\kappa$) increases the quality ($y$) and decreases the price ($p$) chosen in equilibrium.

**Proposition 6** *In a market with $n \geq 2$ media firms, each of which owns $\kappa \geq 1$ purely advertising funded media outlets, the level of quality is relatively lower and prices are relatively higher than if there were $\kappa n$ separate firms.*

As to be expected, due to the increasing monopoly power, if there are $n$ firms with $\kappa > 1$ outlets rather than $\kappa n$ separate firms, then the level of quality ($y$) becomes relatively lower, and, in the cases where prices are positive, they are relatively higher. In the remainder of the paper, to simplify, we treat the case $\kappa = 1$.

4 Non-commercial media

In order to capture the role of media outlets such as non-profit radio stations, TV stations, newspapers, and to some extent also the increasingly
important internet with all its websites and weblogs, many of which are in direct competition with other mainstream outlets, we next consider a second type of media outlet, which we refer to as non-commercial. Before studying the case of mixed markets with commercial and non-commercial outlets, we first briefly consider the case with only non-commercial ones.

We assume non-commercial media maximize the utility they provide to their audience, $u_i$, and operate under a given budget $B_i \geq 0$. While we do not intend to model public broadcasting in this paper (see e.g., Armstrong, 2005, and Armstrong and Weeds, 2005, 2007, on this), we do not exclude that this type of modeling may in fact capture some public media outlets.\footnote{In this framework, since market shares are determined as \[ s_i = \frac{A_m}{m} + \frac{1}{N(N-1)} \sum_{j \neq i} (u_i - u_j), \] where $A_m$ is defined just as in Eq. (1), they can be written as $s_i = \phi_0 + \phi_1 u_i$, for $\phi_0, \phi_1 > 0$ constant as far as $i$’s optimization problem is concerned, and taking the level of utility of the other outlets as given, we see that maximizing $u_i$ is equivalent to maximizing its market share $s_i$.}

The optimization problem of such a non-commercial outlet is therefore

$$\max_{x_i, y_i, p_i} \quad \text{subject to} \quad \frac{\delta}{2} y_i^2 \leq B_i + s_i p_i,$$

where $B_i \geq 0$ is the outlet’s budget. The solution to this is given by

$$x_i = 1, \quad y_i = \sqrt{\frac{2(B_i + s_i p_i)}{\delta}}, \quad p_i = \left[ \frac{\beta^2 s_i}{2\delta} - \frac{B_i}{s_i} \right]^+,$$

where the optimal price charged is computed from the corresponding FOC,

$$\frac{\partial u_i}{\partial p_i} = \frac{\beta s_i}{2\delta} \frac{1}{\sqrt{\frac{B_i + s_i p_i}{\delta}}} - 1 = 0.$$

In particular, assuming a situation where all media outlets are symmetric and non-commercial, we get the following:

$$x = 1, \quad y = \sqrt{\frac{2(B + sp)}{\delta}}, \quad p = \left[ \frac{\beta^2 s}{2\delta} - \frac{B}{s} \right]^+,$$

where here $s = \frac{A_m}{m} = \frac{2N-m-1}{N(N-1)}$.\footnote{In this framework, since market shares are determined as $s_i = \frac{A_m}{m} + \frac{1}{N(N-1)} \sum_{j \neq i} (u_i - u_j)$, where $A_m$ is defined just as in Eq. (1), they can be written as $s_i = \phi_0 + \phi_1 u_i$, for $\phi_0, \phi_1 > 0$ constant as far as $i$’s optimization problem is concerned, and taking the level of utility of the other outlets as given, we see that maximizing $u_i$ is equivalent to maximizing its market share $s_i$.}
Proposition 7 In a market with $N$ potential media outlets and $m < N$ symmetric purely non-commercial media outlets there is a unique equilibrium with maximum accuracy ($x = 1$) for any $m \geq 1$. The allocated budget and audience revenue (when these are positive) are spent entirely on providing quality, $y = \sqrt{\frac{2(B + s)p}{\delta}}$, and prices are given by $p = \left[ \frac{\beta^2 s}{2s} - \frac{B}{s} \right]^+$. Notice that, when prices are zero, the quality level is independent of the number (and strategies) of the other non-commercial outlets, whereas, if prices are positive, we can write the level of quality as

$$y = \sqrt{\frac{B}{\delta} + \frac{1}{2} \left( \frac{\beta(2N - m - 1)}{\delta N(N - 1)} \right)^2},$$

so that, the more non-commercial media outlets there are (higher $m$), the lower the quality provided. This is due to the fact that prices and market shares are decreasing with $m$, for given $N$.

5 Mixed media markets

Clearly, most markets have commercial and non-commercial media outlets operating simultaneously. We next consider this case and derive equilibrium coverage and prices when media outlets act symmetrically relative to their type. We also characterize actual equilibrium shares of commercial vs. non-commercial media. For simplicity, we do not allow media firms to own more than one outlet.

There are $N$ potential media outlets of which $n \geq 2$ are commercial and $m \geq 0$ are non-commercial media outlets, where $n + m < N$ such that there are $N - n - m > 0$ potential firms that are not present in the market. As before, commercial outlets are indexed by $i = 1, \ldots, n$, non-commercial ones are indexed $i = n + 1, \ldots, n + m$, and the potential ones not present in the market by $i = n + m + 1, \ldots, N$. Let $A_{n+m}$ denote the total share of the commercial and non-commercial outlets’ audience, we can write:

$$A_{n+m} = \frac{2N - n - m - 1}{N(N - 1)}(n + m).$$
Let \( \sigma = \sum_{i=1}^{n} s_i \) be the actual share of consumers that access commercial media outlets from among all consumers, so that \( A_{n+m} - \sigma \) is the share that access non-commercial media outlets.

The assumption that viewers have a preference for only two outlets immediately implies that \( 0 \leq (N-m)(N-m-1) \leq \sigma \leq 1 - \frac{(N-n)(N-n-1)}{N(N-1)} \leq 1 \) with the first and last inequalities strict if \( n, m > 0 \).

From Section 4, assuming budgets are not too small or there are sufficiently many non-commercial outlets, we have \( p_{NC} = 0 \) and can write the optimal strategy of the non-commercial media as

\[
x_{NC} = 1, \ y_{NC} = \sqrt{\frac{2B}{\delta}}; \ p_{NC} = 0.
\]

We can write the profit of the commercial media, for \( i = 1, \ldots, n \), as

\[
\pi_i = s_i \eta C + s_i p_i - \frac{\delta}{2} y_i^2,
\]

where, since \( x_{NC} = 1 \), we have,

\[
C = C_0 \cdot e^{-\psi(\sum_{j=1}^{n} s_j x_j + \sum_{j=n+1}^{n+m} s_j)}.
\]

(Notice that \( \sigma \) depends on the shares of all firms and cannot be therefore treated as a constant.) Recall, for \( i = 1, \ldots, n \),

\[
s_i = \frac{n + m - 1}{N(N-1)} + \frac{1}{N(N-1)t} \sum_{j \neq i} (u_i - u_j) + \frac{2(N-n-m)}{N(N-1)}
\]

\[
= \frac{A_{n+m}}{n+m} + \frac{n(u_i - \bar{u} C)}{N(N-1)t} + \frac{m(u_i - \bar{u}_{NC})}{N(N-1)t}.
\]

(7)

Computing marginal profits, assuming type-symmetry, gives,

\[
\frac{\partial \pi}{\partial x_i} = \left( \frac{\alpha(n + m - 1)}{N(N-1)t} - \frac{\psi \sigma}{n} \left( \frac{\sigma}{n} - \frac{\alpha m (1-x)}{N(N-1)t} \right) \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1-x))} + \frac{\alpha(n+m-1)}{N(N-1)t} p_C
\]

\[
\frac{\partial \pi}{\partial y_i} = \left( \frac{\beta(n + m - 1)}{N(N-1)t} + \frac{\psi \sigma \beta m (1-x)}{n N(N-1)t} \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1-x))} + \frac{\beta(n+m-1)}{N(N-1)t} p_C - \delta y C
\]

\[
\frac{\partial \pi}{\partial p_i} = -\left( \frac{n + m - 1}{N(N-1)t} + \frac{\psi \sigma m (1-x)}{n N(N-1)t} \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1-x))} + \frac{\sigma}{n} \frac{n + m - 1}{N(N-1)t} p_C.
\]
Solving for $x_C$ yields

$$x_C = 1 + \frac{(n + m - 1)n}{\sigma \psi m} - \frac{\sigma t N(N - 1)}{\alpha m n}, \quad (8)$$

and further solving for $(y_C, p_C)$ yields

$$
\begin{align*}
p_C &= \frac{\sigma N(N - 1)t}{n(n + m - 1)} - \left(1 + \frac{\sigma \psi m(1 - x)}{n(n + m - 1)}\right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1 - x))} = 0 \\
y_C &= \frac{\beta}{\delta N(N - 1)t} \left(n + m - 1 + \frac{\sigma \psi m(1 - x)}{n}\right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1 - x))},
\end{align*}
$$

where it can be checked that again both SOC’s are satisfied. Therefore, from Eq. (8) and using the bounds $\frac{n}{N} \leq \sigma \leq \frac{2n}{N}$, and defining

$$
\Lambda_0(\psi, t, m, n, N) = \frac{\psi t(N - 1)}{2 \psi m + (n + m - 1)N} \left(\leq \frac{\sigma^2 \psi t N(N - 1)}{n(\sigma \psi m + n(n + m - 1))}\right)
$$

and

$$
\Lambda_1(\psi, t, m, n, N) = \frac{4 \psi t(N - 1)}{(n + m - 1)N} \left(\geq \frac{\sigma^2 \psi t N(N - 1)}{n^2(n + m - 1)}\right)
$$

as upper and lower bounds for $\alpha$, we can distinguish the following three cases:

CASE 1: $x = 0$ occurs when $\alpha \leq \Lambda_0(\psi, t, m, n, N)$;
CASE 2: $x \in [0, 1]$ occurs when $\Lambda_0(\psi, t, m, n, N) < \alpha < \Lambda_1(\psi, t, m, n, N)$;
CASE 3: $x = 1$ occurs when $\alpha \geq \Lambda_1(\psi, t, m, n, N)$.

Depending on the values of the parameters, any of the three cases can occur. It is clear, already from Eq. (8), that larger values of $n, m$ tend to increase $x$. Here, given $\alpha$ and the other parameters, larger values of $n, m$ tend to make the conditions for CASE 3 more likely to be satisfied. Hence, more commercial and non-commercial outlets in the market tend to increase the level of accuracy. We now consider the first and third cases in more detail. (We omit CASE 2 as the set of parameters under which it obtains is relatively small; it is also tedious to analyze and is anyways intermediate between the other two cases.)

--

5Since $\sigma$ is not (yet) determined and depends in an important way on $n$, we do not solve for $n$ here, but rather for $\alpha$ which we approximate by the functions $\Lambda_0$ and $\Lambda_1$. 

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CASE 1: $x = 0$. Here we have

$$x_C = 0, y_C = \frac{\beta\eta C_0 e^{-\psi (A_{nm} - \sigma)}}{\delta N(N - 1) t} \left( n + m - 1 + \frac{\sigma \psi m}{n} \right), p_C = 0$$

and

$$x_{NC} = 1, y_{NC} = \sqrt{\frac{2B}{\delta}}, p_{NC} = 0.$$  

Which leads to

$$u_C = \beta y_C \text{ and } u_{NC} = \alpha + \beta y_{NC},$$

so that, with $\sigma = \frac{n}{N(N - 1) t} \left( (2N - n - m - 1) t \right. + \beta m \left( \frac{\beta \eta C_0 e^{-\psi (A_{nm} - \sigma)}}{\delta N(N - 1) t} \left( n + m - 1 + \frac{\sigma \psi m}{n} \right) - \sqrt{\frac{2B}{\delta}} \right) \frac{nm \alpha}{N(N - 1) t}$,

CASE 3: $x = 1$. Here we have

$$x_C = 1, y_C = \frac{\beta \eta (n + m - 1) C_0 e^{-\psi A_{nm}}}{\delta N(N - 1) t}, p_C = 0.$$
and

$$x_{NC} = 1, \ y_{NC} = \sqrt{\frac{2B}{\delta}}, \ p_{NC} = 0.$$  

This leads to

$$u_C = \alpha + \beta y_C \ \text{and} \ u_{NC} = \alpha + \beta y_{NC}$$

so that, with

$$\alpha = \frac{2N-n-m-1}{N(N-1)} + \frac{m(u_C-u_{NC})}{N(N-1)t},$$

we have

$$\sigma = \frac{n}{N(N-1)} \left(2N - n - m - 1 + \frac{\beta m}{t} \left(\frac{\beta \eta(n + m - 1)C_0 e^{-\psi A_{nm}}}{\delta N(N - 1)t} - \sqrt{\frac{2B}{\delta}}\right)\right).$$

Finally, substituting this expression for $\sigma$ into Eq. (8) above and solving for $n$ gives a new expression for $\bar{n}_m$, the number of (separately owned) commercial media outlets that will guarantee that there will be no censorship given that there are $m$ non-commercial media outlets in the market. Clearly, the more non-commercial media outlets there are, the smaller $\bar{n}_m$ has to be. (See Figure 5.)

In general we can summarize as follows.

**Proposition 8** In a market with $N$ potential media outlets of which $n$ are commercial and $m$ are non-commercial, and where $N > n+m$, we distinguish three possible situations:
(1) \( x_C = 0 \) when \( \alpha \leq \Lambda_0(\psi, t, m, n, N) \),
(2) \( x_C \in [0, 1] \) when \( \Lambda_0(\psi, t, m, n, N) \leq \alpha \leq \Lambda_1(\psi, t, m, n, N) \),
(3) \( x_C = 1 \) when \( \alpha \geq \Lambda_1(\psi, t, m, n, N) \).

In particular, accuracy and quality of the commercial media (respectively \( x_C \) and \( y_C \)) tend to increase with higher values of \( m \) and \( n \) and tend to decrease with lower values of \( N \) and \( t \).

Moreover, everything else equal, an increase in base consumption \((C_0)\) increases the actual share of commercial media \((\sigma)\), while an increase in the budgets of non-commercial media \((B)\) decreases it.

An increase in the budget allocated to non-commercial media can partially crowd out commercial media, while an increase in base consumption \(C_0\) of branded products can also partially crowd out non-commercial media.\(^6\)

Notice also that in Case (1) above \((x_C = 0, \text{minimum accuracy})\) the level of quality of the commercial media outlets \((y_C)\) is higher than in Case (3) \((x_C = 1, \text{maximum accuracy})\) due to the fact that more quality is needed to offset the lower level of reporting \(x_C\). At the same time, an increase in the non-commercial media outlets’ budget \(B\) tends to (decrease \(\sigma\) and therefore) increase \(y_C\) in Case (1) and has no effect in Case (3); it can be shown that the level of quality \((y_C)\) also increases with the number of non-commercial media outlets \((m)\).

Overall, our analysis seems to indicate that even with very low budgets, non-commercial media can have beneficial effects for both accuracy of content \((x_C)\) and quality \((y_C)\) of the commercial media. We should stress that the spokes model has an exogenous symmetry in terms of the positioning of the (actual and potential) outlets, which possibly gives too much audience to the non-commercial outlets. In this sense, while there are, for example, clearly very many non-commercial websites that might qualify as a “media outlet” in our sense, only very few seem to have the capacity and visibility to count as an actual “media outlet” in our underlying spokes model. (\(\rightarrow\) data on rankings and numbers of visits of websites.)

\(^6\)Notice that when taking the limits \(C_0, B \to \infty\) leads to violations of the running assumption, \(|u_i - u_j| < t\), in which case the corresponding limiting values for \(\sigma\) have to be calculated separately.
6 Free entry -- incomplete

To allow for free entry into the above markets, consider the setting with $N$ potential firms, $n$ commercial firms (with $\kappa = 1$) and no non-commercial firms. We assume a fixed cost $K > 0$ to operate any given media outlet and solve for the level of fixed costs $\bar{K}$ that supports fully informative equilibria, that is, $n > \bar{n}$ firms in equilibrium if $K < \bar{K}$.

Recall, that in the case of purely commercial media with $n \geq \bar{n}$, we have, $x = 1, y = \frac{\beta(n-1)}{N(N-1)t} \eta C_0 e^{-\psi A}, p = 0$ so that, as the lower bound for the fixed costs supporting fully informative equilibria, we get

$$\bar{K} = A \frac{n \eta C_0 e^{-\psi A}}{N} - \frac{\delta}{2} \left( \frac{\beta(n-1)}{\delta N(N-1)t} \eta C_0 e^{-\psi A} \right)^2$$

$$= \frac{\eta C_0 e^{-\psi A}}{N(N-1)} \left( 2N - \bar{n} - 1 - \frac{\beta^2(n-1)^2}{2\delta t^2} \eta C_0 e^{-\psi A} \right),$$

where $\bar{n} = 2N - 1 + \alpha \frac{N(N-1)}{2\delta t} - \sqrt{\alpha N(N-1)^2(\alpha N + 8\psi t)}$. If actual fixed costs are substantially above the obtained $\bar{K}$, then fully informative market structures will not be supported. Clearly, the larger $\bar{n}$ the smaller $\bar{K}$ will be, and the less likely it will be that a fully informative market structure can be supported.

7 Welfare analysis -- incomplete

We compute total utility in the following two cases:

CASE 1: $[N; (n, \kappa)]$ or $n$ commercial media firms ($n < N$) with $\kappa$ outlets each:

$$W_{\kappa n} = (\alpha x_C + \beta y_C) \left( \frac{\kappa n}{N} + \frac{\kappa n(N - \kappa n)}{N(N-1)} \right) - \left( \frac{\kappa n}{N} \frac{1}{4} + \frac{\kappa n(N - \kappa n)}{N(N-1)} \frac{3}{4} \right)$$

$$= (\alpha x_C + \beta y_C - \frac{1}{4}) \frac{\kappa n}{N} + \left( \alpha x_C + \beta y_C - \frac{3}{4} \right) \frac{\kappa n(N - \kappa n)}{N(N-1)},$$

where $x_C = 0 \ (= 1)$ if $n < (\geq) \bar{n}(\kappa)$ and $y_C = \frac{\beta(n-1)}{\delta N(N-1)t} \eta C_0 e^{-\psi A_\kappa x_C}$. Assuming that $n$ is not close to $\bar{n}(\kappa)$ and $\kappa n$ is not close to $N$, then it seems clear that increasing both $\kappa$ and $n$ increases welfare; through increases in
and since neither decrease $x_C$. On the other hand, as mentioned above, one should be cautious about increasing $\kappa$ in the other cases as it may force the total number of owners below the cutoff $\hat{n}(\kappa)$ if the constraint $n \leq \frac{N}{\kappa}$ is binding.

The optimal ownership structure, assuming a given target number of total outlets of say $M$ (which in the first best case is equal to $N$) is the one that has the most number of different firms/owners $n$ with $\kappa$ outlets each, satisfying $\kappa = \frac{N}{n}$ and the firms are viable (i.e., cover fixed cost). This is true whether the fixed costs apply at the firm level or at the outlet level.

CASE 2: $[N; (n, 1); (m, 1)]$ or $n$ commercial media firms and $m$ non-commercial media firms ($n + m < N$) with 1 outlet each:

$$W_{mn} = (\alpha x_C + \beta y_C)\sigma + (\alpha + \beta y_{NC}) (A_{mn} - \sigma)$$

$$- \left( \frac{n}{N^4} + \frac{(m + n)(N - m - n)}{N(N - 1)} \frac{3}{4} + m \left( \frac{A_{mn}}{N} - \nu \right) \frac{1}{2} + m\nu \frac{1}{2} \right)$$

$$= \left( \alpha x_C + \beta y_C - \frac{1}{4} \right) \frac{n}{N} + \left( \alpha x_C + \beta y_C - \frac{3}{4} \right) \frac{n(N - m - n)}{N(N - 1)} + \left( \alpha x_C + \beta y_C - \frac{1}{2} \right) m\nu + \left( \alpha + \beta y_{NC} - \frac{A_{mn}}{N} - \nu \right) m\left( \frac{A_{mn}}{N} - \nu \right)$$

$$+ \left( \alpha + \beta y_{NC} - \frac{3}{4} \right) \frac{m(N - m - n)}{N(N - 1)},$$

where $\nu = \frac{\sigma}{m n} - \frac{A_{mn}}{m N}$ is the measure of consumers located on a spoke of any non-commercial media outlet that consumes commercial media outlets; and $x_C = 0$ ($= 1$) if $\alpha \leq \Lambda_0(\psi, t, m, n, N)$ ($\alpha \geq \Lambda_1(\psi, t, m, n, N)$), $y_C = \frac{\beta}{\sigma N(N-1)} \left( n + m - 1 + \frac{\sigma \psi m(1-x)}{n} \right) e^{-\psi (A_{mn} - \sigma(1-x))}$, and $y_{NC} = \sqrt{\frac{2B}{\sigma}}$.

Assuming standard parameter values and $m + n$ is fixed, then the ownership structure $(m, n)$ and distribution of total resources $TB$ that maximizes $W_{mn}$ is the one that has as many commercial outlets as are profitable given fixed costs and all the rest non-commercial outlets, and where the total budget is divided equally across all the non-commercial outlets. In particular (unless a “small” budget may help) an extra commercial media outlet
to be viable, which we exclude as being “rare”\(^7\) the total budget is best spent (evenly) on the non-commercial media outlets since they convert every amount received into raising their quality \(y_{NC}\).

Clearly, the welfare measures used only concern the welfare of media viewers (defined narrowly through their “media utilities \(u_i\)) and so do not take into account the welfare of the media outlets themselves (nor of the advertisers). One way to address this (at least partially) is to compare their profits with an “outside option” amount rather than just with the fixed costs.

Another aspect that is not accounted for by our welfare measure is that the effect on \(x\) may generate important externalities such as having latent problems like global warming not being taken seriously by the public due to low coverage (\(x_C\) small).

### 8 Policy discussion -- incomplete

The results have clear implications for the debate on the optimal ownership structure in media markets. This has become particularly important recently in the US where the FCC has tried twice recently (in 2003 and 2007) to relax ownership rules to allow for media conglomerates to eventually own more outlets. At the same time, the media market in the US is fairly concentrated already.\(^8\) Our results show that excessive concentration of ownership can lead to substantial bias, and, moreover, the numbers we obtain as thresholds for the occurrence of substantial bias are potentially alarming. Clearly, more empirical work is needed to validate the picture sketched by the present model; the stylized facts and insights derived from the model can all in principle be tested by the data. The case of the reporting on global warming

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\(^7\)This occurs roughly when \(\alpha + m \beta \sqrt{\frac{2TB}{\delta m}} < \beta y_C\), which is obtained by comparing \(m\) non-commercial media outlets with \(TB\) split among them and \(n\) commercial outlets with \(m - 1\) non-commercial outlets with (roughly) zero budget and \(n + 1\) commercial outlets, with the budget used essentially to keep them all viable.

\(^8\)According to Bagdikian, 2004, five media conglomerates (Time Warner, Disney, News Corporation, Viacom, and Bertelsman) produce a majority of all of US media consumption; a figure that was around fifty in the early 1980’s, see Bagdikian (1983). Compaine and Gomery (2000) make some qualifications on these figures; see also Baker (2007).
over the last few decades might be a good place to start.

Appendix -- incomplete

Proofs of Propositions 1–3. Compute for $i \neq j$:

\[
\begin{align*}
\frac{\partial s_i}{\partial x_i} &= \frac{\alpha(n - 1)}{N(N - 1)t}, & \frac{\partial s_i}{\partial y_i} &= \frac{\beta(n - 1)}{N(N - 1)t}, & \frac{\partial s_i}{\partial p_i} &= -\frac{n - 1}{N(N - 1)t} \\
\frac{\partial s_j}{\partial x_i} &= -\frac{\alpha}{N(N - 1)t}, & \frac{\partial s_j}{\partial y_i} &= -\frac{\beta}{N(N - 1)t}, & \frac{\partial s_j}{\partial p_i} &= \frac{1}{N(N - 1)t}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial C}{\partial x_i} &= \left( s_i + \frac{\partial s_i}{\partial x_i} x_i + \sum_{j \neq i} \frac{\partial s_j}{\partial x_i} x_j \right) (-\psi) C_0 e^{-\psi \sum_{i=1}^n s_i x_i} \\
&= \left( s_i + \frac{\alpha(n - 1)}{N(N - 1)t} x_i - \sum_{j \neq i} \frac{\alpha}{N(N - 1)t} x_j \right) (-\psi) C_0 e^{-\psi \sum_{i=1}^n s_i x_i},
\end{align*}
\]

and $\frac{\partial C}{\partial y_i} = \frac{\partial C}{\partial p_i} = 0$. Assuming symmetric equilibrium with $x_i = x, y_i = y, p_i = p$ for all $i$, we can write:

\[
\frac{\partial C}{\partial x_i} = \frac{A_n}{n} (-\psi) C_0 e^{-\psi A_n x} = \frac{2N - n - 1}{N(N - 1)} (-\psi) C_0 e^{-\psi \frac{2N^2 - n - 1}{N(N - 1)} N x},
\]
Marginal profits under symmetry are:

\[
\frac{\partial \pi_i}{\partial x_i} = \eta \left( \frac{\partial s_i}{\partial x_i} C + s_i \frac{\partial C}{\partial x_i} \right) + p_i \frac{\partial s_i}{\partial x_i} \\
= \eta \left( \frac{\alpha(n-1)}{N(N-1)t} C_0 e^{-\psi A_{nx}} + \frac{A_n}{n} \frac{A_n}{n} (-\psi) C_0 e^{-\psi A_{nx}} \right) + p \frac{\alpha(n-1)}{N(N-1)t} \\
= \eta \left( \frac{\alpha(n-1)}{N(N-1)t} - \frac{\psi A_{nx}^2}{n^2} \right) \eta C_0 e^{-\psi A_{nx}} + p \frac{\alpha(n-1)}{N(N-1)t}
\]

\[
\frac{\partial \pi_i}{\partial y_i} = \eta \left( \frac{\partial s_i}{\partial y_i} + s_i \frac{\partial C}{\partial y_i} \right) + p_i \frac{\partial s_i}{\partial y_i} - \delta y_i \\
= \eta \left( \frac{\beta(n-1)}{N(N-1)t} C_0 e^{-\psi A_{nx}} + p \frac{\beta(n-1)}{N(N-1)t} - \delta y \right) \\
\frac{\partial \pi_i}{\partial p_i} = \eta \left( \frac{\partial s_i}{\partial p_i} C + s_i \frac{\partial C}{\partial p_i} \right) + \left( s_i + p_i \frac{\partial s_i}{\partial p_i} \right) \\
= \eta \left( \frac{-\beta(n-1)}{N(N-1)t} C_0 e^{-\psi A_{nx}} + \left( \frac{A_n}{n} + p \frac{\beta(n-1)}{N(N-1)t} \right) \right) \\
= -\eta \frac{n-1}{N(N-1)t} C_0 e^{-\psi A_{nx}} + \frac{A_n}{n} - \frac{n-1}{N(N-1)t}
\]

Proof of Proposition 1. By inspection of the marginal profits with respect to \( x_i \), we have that profits are increasing whenever \( \alpha(n-1) > \frac{\psi t(2N-n-1)^2}{N(N-1)} \) and decreasing whenever \( \alpha(n-1) < \frac{\psi t(2N-n-1)^2}{N(N-1)} \). This means that when the first inequality holds, then it is best to set the maximum level of accuracy \( (x = 1) \), while it is best to set the minimum level \( (x = 0) \) whenever the second inequality holds. In other words, solving for the equality

\[
\frac{\alpha(n-1)}{N(N-1)t} = \frac{\psi A_{nx}^2}{n^2} \iff \frac{\alpha(n-1)}{t} = \frac{\psi(2N-n-1)^2}{N(N-1)}
\]

for \( n \), gives us the number (of media outlets)

\[
\bar{n} = 2N - 1 + \frac{\alpha N(N-1)}{2\psi t} - \frac{\sqrt{\alpha N(N-1)^2(\alpha N + 8\psi t)}}{2\psi t}
\]

\[
= 2N - 1 - \frac{N-1}{2\psi t} \left( \sqrt{(\alpha N)^2 + \alpha N8\psi t} - \alpha N \right),
\]

at which the optimal strategy switches from minimum to maximum accuracy. It can be shown that for \( n < \bar{n} \) we have \( x = 0 \) as the unique solution; while for \( n > \bar{n} \) we have \( x = 1 \) as the unique solution. We can state the following.
Proof of Proposition 3. From the symmetric FOC’s we immediately get $x = 1$ and for the quality and price we get
\[
\frac{\partial \pi}{\partial y} = p \frac{\beta(n - 1)}{N(N - 1)t} - \delta y = 0, \quad \frac{\partial \pi}{\partial p} = \frac{A_n}{n} - p \frac{n - 1}{N(N - 1)t} = 0,
\]
where $A_n$ is defined in Eq. (1). Solving for $p$ then for $y$, leads to
\[
p = \frac{A_n N(N - 1)t}{n(n - 1)} = \frac{(2N - n - 1)t}{n - 1} \quad \text{and} \quad y = \frac{\beta(2N - n - 1)}{\delta N(N - 1)}
\]
as the equilibrium price and quality.

Proposition 4: The FOC’s for the general case are
\[
\begin{align*}
\frac{\partial \pi_i}{\partial x_i} &= \eta \left( \frac{\alpha(n - 1)}{N(N - 1)t} - \frac{\psi A_n^2}{n^2} \right) C_0 e^{\psi A_n x} + p \frac{\alpha(n - 1)}{N(N - 1)t} \\
\frac{\partial \pi_i}{\partial y_i} &= \eta \left( \frac{\beta(n - 1)}{N(N - 1)t} C_0 e^{\psi A_n x} \right) + p \frac{\beta(n - 1)}{N(N - 1)t} - \delta y \\
\frac{\partial \pi_i}{\partial p_i} &= \eta \left( \frac{-(n - 1)}{N(N - 1)t} C_0 e^{\psi A_n x} \right) + \left( \frac{A_n}{n} + p \frac{-(n - 1)}{N(N - 1)t} \right)
\end{align*}
\]
Solving for $(x, y, p)$ yields in particular,
\[
p = \left[ \frac{(2N - n - 1)t}{n - 1} - \eta C_0 e^{\psi A_n x} \right]^+, \quad y = \frac{\beta(n - 1)}{\delta N(N - 1)} \eta C_0 e^{\psi A_n x}.
\]

Proof of Proposition 5. Defining total audience reached as
\[
A_{kn} = \frac{(2N - \kappa n - 1)\kappa n}{N(N - 1)},
\]
we can write for $j \neq \kappa_i$,
\[
\begin{align*}
\frac{\partial s_{\kappa_i}}{\partial x_{\kappa_i}} &= \frac{\alpha(n\kappa - 1)}{N(N - 1)t}, \quad \frac{\partial s_{\kappa_i}}{\partial y_{\kappa_i}} = \frac{\beta(n\kappa - 1)}{N(N - 1)t}, \quad \frac{\partial s_{\kappa_i}}{\partial p_{\kappa_i}} = -\frac{n\kappa - 1}{N(N - 1)t} \\
\frac{\partial s_j}{\partial x_{\kappa_i}} &= -\frac{\alpha}{N(N - 1)t}, \quad \frac{\partial s_j}{\partial y_{\kappa_i}} = -\frac{\beta}{N(N - 1)t}, \quad \frac{\partial s_j}{\partial p_{\kappa_i}} = \frac{1}{N(N - 1)t}
\end{align*}
\]
\[
\frac{\partial C}{\partial x_{i_i''}} = \left( s_{i_i''} + \frac{\partial s_{i_i''}}{\partial x_{i_i''}} x_{i_i''} + \sum_{j \neq i_i''} \frac{\partial s_j}{\partial x_{i_i''}} x_j \right) (-\psi) C_0 \cdot e^{-\psi \sum_{i_i=1} s_{i_i''} x_{i_i''}}.
\]

Notice also that \( \frac{\partial C}{\partial y_{i_i''}} = \frac{\partial C}{\partial p_{i_i''}} = 0 \).

From this we compute the FOC's.

\[
\frac{\partial \pi}{\partial x_{i_i''}} = \frac{\partial}{\partial x_{i_i''}} \left( s_{i_i''} \eta C - \frac{\delta}{2} y_{i_i''}^2 \right) + \sum_{i_i' \neq i_i''} \frac{\partial}{\partial x_{i_i''}} \left( s_{i_i'} \eta C - \frac{\delta}{2} y_{i_i'}^2 \right).
\]

\[
\frac{\partial \pi}{\partial x_{i_i''}} = \eta \left( \frac{\partial s_{i_i''}}{\partial x_{i_i''}} C + s_{i_i''} \frac{\partial C}{\partial x_{i_i''}} \right) + \sum_{i_i' \neq i_i''} \eta \left( \frac{\partial s_{i_i'}}{\partial x_{i_i''}} C + s_{i_i'} \frac{\partial C}{\partial x_{i_i''}} \right) C_0 e^{-\psi A_{i_n} x}.
\]

Again, this expression is either positive or negative depending on whether \( n \) is smaller or greater than \( \bar{n}(\kappa) \) given by:

\[
\bar{n}(\kappa) = \frac{2N - 1}{\kappa} + \frac{\alpha \kappa (N - 1)}{2\psi \kappa^2} - \sqrt{\alpha \kappa (N - 1)(\alpha \kappa N(1) + 4\psi \kappa(2N - \kappa - 1))t} \]

\[
= \frac{2N - 1}{\kappa} - \sqrt{\alpha \kappa (N - 1)^2 + 4\psi \kappa t\alpha \kappa N(1)(2N - \kappa - 1) - \alpha \kappa (N - 1)}.
\]

**Proof of Proposition 6.** As before we have, \( \frac{\partial C}{\partial y_{i_i''}} = \frac{\partial C}{\partial p_{i_i''}} = 0 \), from which
we can compute:

\[
\frac{\partial \pi}{\partial y_{\kappa_i}} = \frac{\partial}{\partial y_{\kappa_i}} \left( s_{\kappa_i}'' \eta C + s_{\kappa_i}'' p_{\kappa_i} - \frac{\delta}{2} y_{\kappa_i}^2 \right) + \sum_{\kappa_i' \neq \kappa_i} \frac{\partial}{\partial y_{\kappa_i'}} \left( s_{\kappa_i'}'' \eta C + s_{\kappa_i'}'' p_{\kappa_i'} - \frac{\delta}{2} y_{\kappa_i'}^2 \right)
\]

\[
= \left( \frac{\partial s_{\kappa_i}''}{\partial y_{\kappa_i}} (\eta C + p_{\kappa_i}'') + \eta s_{\kappa_i}'' \frac{\partial C}{\partial y_{\kappa_i}''} - \delta y_{\kappa_i}'' \right) + \sum_{\kappa_i' \neq \kappa_i} \left( \frac{\partial s_{\kappa_i'}''}{\partial y_{\kappa_i'}} (\eta C + p_{\kappa_i'}'') + \eta s_{\kappa_i'}'' \frac{\partial C}{\partial y_{\kappa_i}''} \right)
\]

\[
= \left( \frac{\beta (\kappa n - 1)}{N(N - 1)t} + \sum_{\kappa_i' \neq \kappa_i} \frac{(-\beta)}{N(N - 1)t} \right) \left( \eta C_0 e^{-\psi A_{\kappa_i} x} + p \right) - \delta y_{\kappa_i}''
\]

\[
\frac{\partial \pi}{\partial p_{\kappa_i}''} = \frac{\partial}{\partial p_{\kappa_i}''} \left( s_{\kappa_i}'' \eta C + s_{\kappa_i}'' p_{\kappa_i} - \frac{\delta}{2} y_{\kappa_i}^2 \right) + \sum_{\kappa_i' \neq \kappa_i} \frac{\partial}{\partial p_{\kappa_i'}} \left( s_{\kappa_i'}'' \eta C + s_{\kappa_i'}'' p_{\kappa_i'} - \frac{\delta}{2} y_{\kappa_i'}^2 \right)
\]

\[
= \left( s_{\kappa_i}'' + \frac{\partial s_{\kappa_i}''}{\partial p_{\kappa_i}} (\eta C + p_{\kappa_i}'') + \eta s_{\kappa_i}'' \frac{\partial C}{\partial p_{\kappa_i}''} \right) + \sum_{\kappa_i' \neq \kappa_i} \left( \frac{\partial s_{\kappa_i'}''}{\partial p_{\kappa_i'}} (\eta C + p_{\kappa_i'}'') + \eta s_{\kappa_i'}'' \frac{\partial C}{\partial p_{\kappa_i}''} \right)
\]

\[
= \frac{A_{\kappa n}}{\kappa n} \frac{\kappa(n - 1)}{N(N - 1)t} \eta C_0 e^{-\psi A_{\kappa_i} x} - \frac{\kappa n - 1}{N(N - 1)t} p_{\kappa_i}'' + \sum_{\kappa_i' \neq \kappa_i} \frac{1}{N(N - 1)t} p_{\kappa_i'}.
\]

Under symmetry, we have:

\[
p = \left[ \frac{2(N - \kappa n - 1)t}{\kappa(n - 1)} - \eta C_0 e^{-\psi A_{\kappa_i} x} \right]^+ = 0, \quad y = \frac{\beta \kappa(n - 1)}{\delta N(N - 1)t} \eta C_0 e^{-\psi A_{\kappa_i} x}
\]

**Proof of Proposition 8.** Recall, for \(i = 1, \ldots, n\),

\[
s_i = \frac{n + m - 1}{N(N - 1)} + \frac{1}{N(N - 1)t} \sum_{j \neq i} (u_i - u_j) + \frac{2(N - n - m)}{N(N - 1)}
\]

\[
= \frac{A_{n+m}}{n + m} \frac{n(u_i - \bar{u}_C)}{N(N - 1)t} + m(u_i - \bar{u}_{NC}) \frac{N(N - 1)}{N(N - 1)t}.
\]

Moreover, for \(i \neq j\),

\[
\frac{\partial s_i}{\partial x_i} = \frac{\alpha(n + m - 1)}{N(N - 1)t}, \quad \frac{\partial s_i}{\partial y_i} = \frac{\beta(n + m - 1)}{N(N - 1)t}, \quad \frac{\partial s_i}{\partial p_i} = \frac{n + m - 1}{N(N - 1)t}
\]

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\[
\frac{\partial s_j}{\partial x_i} = -\frac{\alpha}{N(N-1)t}, \quad \frac{\partial s_j}{\partial y_i} = -\frac{\beta}{N(N-1)t}, \quad \frac{\partial s_j}{\partial p_i} = \frac{1}{N(N-1)t}
\]

\[\frac{\partial C}{\partial x_i} = \left( s_i + \sum_{j \neq i} s_j \frac{\partial s_j}{\partial x_i} + \sum_{j=n+1}^{n+m} \frac{\partial s_j}{\partial x_i} \right) (-\psi) C_0 e^{-\psi \left( \sum_{i=1}^{n} s_i x_i + \sum_{i=n+1}^{n+m} s_i \right)} \]

\[= \left( s_i + \alpha(n + m - 1) \frac{N(N-1)t}{N(N-1)} x_i - \sum_{j \neq i, j=1}^{n} \frac{\alpha}{N(N-1)t} x_j - \frac{m \alpha}{N(N-1)t} \right) (-\psi) C_0 e^{-\psi \left( \sum_{i=1}^{n} s_i x_i + \sum_{i=n+1}^{n+m} s_i \right)} \]

and assuming symmetric equilibrium we have \( x_i = x, y_i = y, p_i = p \) for all \( i = 1, \ldots, n \), so that:

\[\frac{\partial C}{\partial x_i} = \left( \frac{2N - n - m - 1}{N(N-1)} + \frac{m(\bar{u}_C - \bar{u}_{NC})}{N(N-1)t} - \frac{\alpha m(1-x)}{N(N-1)t} \right) (-\psi) C_0 e^{-\psi(\sigma x + 1 - \sigma)} \]

where it can be checked that \( \sigma = \frac{1}{N} + \frac{m(\bar{u}_C - \bar{u}_{NC})}{N(N-1)t} \). Also,

\[\frac{\partial C}{\partial y_i} = \left( \sum_{j \neq i} \frac{\partial s_j}{\partial y_i} + \sum_{j=n+1}^{n+m} \frac{\partial s_j}{\partial y_i} \right) (-\psi) C_0 e^{-\psi \left( \sum_{i=1}^{n} s_i x_i + \sum_{i=n+1}^{n+m} s_i \right)} \]

\[= \frac{\beta m(1-x)}{N(N-1)t} \psi C_0 e^{-\psi(A_{nm} - \sigma(1-x))} \]

\[\frac{\partial C}{\partial p_i} = \left( \sum_{j \neq i} \frac{\partial s_j}{\partial p_i} + \sum_{j=n+1}^{n+m} \frac{\partial s_j}{\partial p_i} \right) (-\psi) C_0 e^{-\psi \left( \sum_{i=1}^{n} s_i x_i + \sum_{i=n+1}^{n+m} s_i \right)} \]

\[= -\frac{m(1-x)}{N(N-1)t} \psi C_0 e^{-\psi(A_{nm} - \sigma(1-x))} \]

Notice that, if \( x = 1 \), then \( \frac{\partial C}{\partial y_i} = 0 \), otherwise, \( \frac{\partial C}{\partial y_i} > 0 \), \( \frac{\partial C}{\partial p_i} < 0 \).
Computing marginal profits, assuming type-symmetry, gives,

\[
\frac{\partial \pi}{\partial x_i} = \eta \left( \frac{\partial s_i}{\partial x_i} C + s_i \frac{\partial C}{\partial x_i} \right) + p_i \frac{\partial s_i}{\partial x_i} = \left( \frac{\alpha(n + m - 1)}{N(N - 1)t} - \frac{\psi \sigma}{n} \left( \frac{\sigma}{n} - \frac{\alpha m(1 - x)}{N(N - 1)t} \right) \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1 - x))} + \frac{\alpha(n + m - 1)}{N(N - 1)t} p_C
\]

\[
\frac{\partial \pi}{\partial y_i} = \eta \left( \frac{\partial s_i}{\partial y_i} C + s_i \frac{\partial C}{\partial y_i} \right) + p_i \frac{\partial s_i}{\partial y_i} - \delta y_i = \left( \frac{\beta(n + m - 1)}{N(N - 1)t} + \frac{\psi \sigma \beta m(1 - x)}{n N(N - 1)t} \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1 - x))} + \frac{\beta(n + m - 1)}{N(N - 1)t} p_C - \delta y_C
\]

\[
\frac{\partial \pi}{\partial p_i} = \eta \left( \frac{\partial s_i}{\partial p_i} C + s_i \frac{\partial C}{\partial p_i} \right) + s_i + p_i \frac{\partial s_i}{\partial p_i} = - \left( \frac{n + m - 1}{N(N - 1)t} + \frac{\psi \sigma m(1 - x)}{n N(N - 1)t} \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1 - x))} + \frac{\sigma}{n} \left( \frac{n + m - 1}{N(N - 1)t} p_C \right)
\]

Solving for \(x_C\) yields

\[
x = 1 + \frac{(n + m - 1)n}{\sigma \psi m} - \frac{\sigma t N(N - 1)}{\alpha m n}.
\]

Writing \(\frac{\partial C}{\partial x_i} = E(-\psi)C_0 e^{-\psi(A_{nm} - \sigma(1 - x))},\) where \(E = \left( \frac{\sigma}{n} - \frac{\alpha m(1 - x)}{N(N - 1)t} \right),\) the SOC’s give:

\[
\frac{\partial^2 \pi}{\partial x_i^2} = \eta \left( \frac{\partial^2 s_i}{\partial x_i^2} C + 2 \frac{\partial s_i}{\partial x_i} \frac{\partial C}{\partial x_i} + s_i \frac{\partial^2 C}{\partial x_i^2} \right) = \left( 0 + \frac{2\alpha(n + m - 1)}{N(N - 1)t} E(-\psi) + \frac{\sigma}{n} (E^2(-\psi)^2 + \frac{\partial E}{\partial x_i}(-\psi)) \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1 - x))}
\]

\[
= \left( -\frac{2\alpha(n + m - 1)}{N(N - 1)t} E + \frac{\sigma}{n} (E^2(-\psi)^2 - \frac{\partial E}{\partial x_i}(-\psi)) \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1 - x))} < 0
\]

since \(\frac{n}{N} \leq \sigma \leq \frac{2n}{N}\) (show these bounds). This means that Eq. (8) gives the unconstrained profit-maximizing level of accuracy for the commercial media. (It can be checked that \(\frac{\partial x}{\partial m}, \frac{\partial x}{\partial n}, \frac{\partial x}{\partial \alpha} \geq 0\) while \(\frac{\partial x}{\partial \sigma} \leq 0.\) double check!)

Further solving for \((y_C, p_C)\) yields

\[
p = \left[ \frac{\sigma N(N - 1)t}{n(n + m - 1)} - \left( 1 + \frac{\sigma \psi m(1 - x)}{n(n + m - 1)} \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1 - x))} \right]^+ = 0
\]

\[
y = \frac{\beta}{\delta N(N - 1)t} \left( n + m - 1 + \frac{\sigma \psi m(1 - x)}{n} \right) \eta C_0 e^{-\psi(A_{nm} - \sigma(1 - x))},
\]

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where it can be checked that again both SOC’s are satisfied. double-check! Therefore, from Eq. (8) and using the bounds $\frac{n}{N} \leq \sigma \leq \frac{2n}{N}$, (show these bounds), and defining:

$$\Lambda_0(\psi, t, m, n, N) = \frac{\psi t (N-1)}{2\psi m + (n+m-1)N} \left( \leq \frac{\sigma^2 \psi t N (N-1)}{n(\sigma \psi m + n(n+m-1))} \right)$$

and

$$\Lambda_1(\psi, t, m, n, N) = \frac{4\psi t (N-1)}{(n+m-1)N} \left( \geq \frac{\sigma^2 \psi t N (N-1)}{n^2(n+m-1)} \right)$$

as upper and lower bounds for $\alpha$, we can distinguish the following three cases (since $\sigma$ is not (yet) determined and depends in an important way on $n$, we do not solve for $n$ here, but rather for $\alpha$ which we approximate by the functions $\Lambda_0$ and $\Lambda_1$):

CASE 1: $x = 0$ occurs when $\alpha \leq \Lambda_0(\psi, t, m, n, N)$;

CASE 2: $x \in [0, 1]$ occurs when $\Lambda_0(\psi, t, m, n, N) < \alpha < \Lambda_1(\psi, t, m, n, N)$;

CASE 3: $x = 1$ occurs when $\alpha \geq \Lambda_1(\psi, t, m, n, N)$.

References


